HW1 due 8pm tonight.
- Please upload a draft, with group names, by 5pm

HW2 due next Tue 8pm

Jeff is out of town next week (Sep 15-21)

11001_2 = 25
10011_2 = 19

Strings \( x \) and \( y \) are distinguished by string \( z \):

\[
xz \in L \quad \text{xor} \quad yz \in L
\]

\[\Rightarrow \delta(s, x) \neq \delta(s, y)\] in every DFA for \( L \)

For some DFA \( M \), if \( S^*(s, x) = S^*(s, y) \) then

For all \( z \in \Sigma^* \), \( xz \in L \iff yz \in L \)

because \( S^*(s, xz) = S^*(s, yz) \)

If there is a string \( z \)

\[s.t. xz \in L \quad \text{xor} \quad yz \in L\]

then for every DFA \( M \)

\[S^*(s, x) \neq S^*(s, y)\]
This is a Fooling set for $L$

Every pair of elements has a distinct suffix

Every DFA for $L$ has at least 5 states.

$L = \{0^n1^m \mid n \geq 0 \}$

Let $F = 0^*$

Let $x$ and $y$ be arbitrary distinct strings in $F$.

Then $x = 0^n$ and $y = 0^m$ for some integers $n \neq m$.

Let $z = 1^n$

Then $xz = 0^n1^n \in L$

But $yz = 0^m1^n \notin L$ because $n \neq m$.

So $z$ distinguishes $x$ and $y$.

So $F$ is a Fooling set for $L$.

Because $F$ is infinite, $L$ cannot be regular.

$L_2 = \text{palindromes} = \{w \mid w = w^R \}$

Let $F = 0^*$

Let $x$ and $y$ be any distinct strings in $F$.

So $x = 0^n$ and $y = 0^m$ where $m \neq n$.

Let $z = 10^n$
Then $xz = 0^n10^n$ is a palindrome
But $yz = 0^n10^n$ is not because $n \neq m$

So $z$ distinguishes $x$ and $y$
So $F$ is a fooling set for $L$.

Because $F$ is infinite, $L$ cannot be regular.

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$L_3 = \{ \text{www} \mid \text{we} \in \Sigma^* \} = \{ \epsilon, 000, 111, 01001001, \ldots \}$
not 0000 or 01010101 or 10...

Let $F = 0^*$

Let $x, y$ be any distinct strings in $F$
Then $x = 0^n$ and $y = 0^m$ for some integers $n \neq m$

Let $z = 10^n10^1$

Then $xz = \underbrace{0^n10^1}_{X} \overbrace{0^1}^Z \underbrace{0^1}_{Y} \in L$
But $yz = \underbrace{0^n10^10^10^n1}_{Y} \in L$ because $n \neq m$

So $z$ distinguishes $x$ and $y$
So $F$ is a fooling set for $L$.

Because $F$ is infinite, $L$ cannot be regular.
Kleene's Theorem: regular = automatic

DFA $\xrightarrow{c, s}$ NFA $\xleftarrow{\text{regex}}$

$\delta(\emptyset, 10100) = \emptyset, 0, +3$

$\delta^{*}(\emptyset, (0+1)^{*}(00+11)(0+1)^{*})$

DFA accepts $w=abc\ldots z$ iff there is a walk

$S \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{\ldots} q_n \in A$

NFA accepts $w=abc\ldots z$ iff the walk

$S \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{\ldots} q_n \in A$

NFA has the following components:

- $Q$ - finite set of states
- $S \in Q$ start
- $A \subseteq Q$ accepting
- $\delta: Q \times \Sigma \rightarrow 2^{Q}$
- $\delta^{*}: Q \times \Sigma^{*} \rightarrow 2^{Q}$

$$
\delta^{*}(q, w) = \bigcup_{q' \in \delta(q, \epsilon)} \delta^{*}(q', x)
$$

$w = \epsilon$

$w = \alpha x$