**Rice’s Theorem.** Let \( \mathcal{L} \) be any set of languages that satisfies the following conditions:
- There is a Turing machine \( Y \) such that \( \text{Accept}(Y) \in \mathcal{L} \).
- There is a Turing machine \( N \) such that \( \text{Accept}(N) \notin \mathcal{L} \).

The language \( \text{AcceptIn}(\mathcal{L}) := \{ \langle M \rangle \mid \text{Accept}(M) \in \mathcal{L} \} \) is undecidable.

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Prove that the following languages are undecidable using Rice’s Theorem:

1. \( \text{AcceptRegular} := \{ \langle M \rangle \mid \text{Accept}(M) \text{ is regular} \} \)
2. \( \text{AcceptIllini} := \{ \langle M \rangle \mid M \text{ accepts the string } \text{ILLINI} \} \)
3. \( \text{AcceptPalindrome} := \{ \langle M \rangle \mid M \text{ accepts at least one palindrome} \} \)
4. \( \text{AcceptThree} := \{ \langle M \rangle \mid M \text{ accepts exactly three strings} \} \)
5. \( \text{AcceptUndecidable} := \{ \langle M \rangle \mid \text{Accept}(M) \text{ is undecidable} \} \)

**To think about later.** Which of the following are undecidable? How would you prove that?

1. \( \text{Accept}\{\varepsilon\} := \{ \langle M \rangle \mid M \text{ accepts only the string } \varepsilon; \text{ that is, } \text{Accept}(M) = \{ \varepsilon \} \} \)
2. \( \text{Accept}\emptyset := \{ \langle M \rangle \mid M \text{ does not accept any strings; that is, } \text{Accept}(M) = \emptyset \} \)
3. \( \text{Accept}\emptyset := \{ \langle M \rangle \mid \text{Accept}(M) \text{ is not an acceptable language} \} \)
4. \( \text{Accept=Reject} := \{ \langle M \rangle \mid \text{Accept}(M) = \text{Reject}(M) \} \)
5. \( \text{Accept}\neq\text{Reject} := \{ \langle M \rangle \mid \text{Accept}(M) \neq \text{Reject}(M) \} \)
6. \( \text{Accept}\cup\text{Reject} := \{ \langle M \rangle \mid \text{Accept}(M) \cup \text{Reject}(M) = \Sigma^* \} \)