Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check “Yes” if the statement is always true and “No” otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking “I don’t know” is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do not need to prove your answer is correct.

   Read each statement very carefully. Some of these are deliberately subtle.

   (a) If zero is odd, then \( 2 + 2 = 5 \).
   (b) For every language \( L \), and for every string \( w \in L \), there is a DFA that accepts \( w \).
   (c) Two languages \( L \) and \( L' \) are regular if and only if \( L \cap L' \) is regular.
   (d) For every language \( L \), the language \( L^* \) is non-empty.
   (e) Every regular language is recognized by an NFA with exactly 374 accepting states.
   (f) If \( L \) does not have a fooling set of size 374, then \( L \) is regular.
   (g) The language \( \{ \epsilon^{374n} \mid n \geq 374 \} \) is regular.
   (h) The language \( \{ \epsilon^{37n} \epsilon^{4n} \mid n \geq 374 \} \) is regular.
   (i) The language \( \{ \epsilon^{3n} \epsilon^{74} \mid n \leq 374 \} \) is regular.
   (j) The empty language is context-free.

2. For any string \( w \in \{0, 1\}^* \), let \( \text{slash}(w) \) be the string in \( \{0, 1, /\}^* \) obtained from \( w \) by inserting a new symbol \( / \) between any two consecutive symbols that are not equal. For example:

\[
\text{slash}(\epsilon) = \epsilon \\
\text{slash}(00000) = 00000 \\
\text{slash}(000110111) = 000/11/0/111
\]

For any language \( L \subseteq \{0, 1\}^* \), let \( \text{slash}(L) = \{ \text{slash}(w) \mid w \in L \} \).

   (a) Draw or describe a DFA that accepts the language \( \text{slash}(\{0, 1\}^*) \).
   (b) Give a regular expression for the language \( \text{slash}(\{0, 1\}^*) \).
   (c) Prove that for any regular language \( L \), the language \( \text{slash}(L) \) is also regular.

(You do not need to justify your answers to parts (a) and (b).)
3. Let $L$ be the language $\{a^c b^c \mid a + b = 2c\}$
   
   (a) **Prove** that $L$ is not a regular language.
   
   (b) Describe a context-free grammar for $L$. (You do not need to justify your answer.)

4. For each of the following languages $L$, give a regular expression that represents $L$ and draw or describe a DFA that recognizes $L$. You do not need to justify your answers.
   
   (a) All strings in $\{0, 1\}^*$ that do not contain either $001$ or $110$ as a substring
   
   (b) All strings in $\{0, 1, 2\}^*$ that do not contain either $01$ or $12$ as a substring

5. For any string $w \in \{0, 1\}^*$, let $\text{obliviate}(w)$ denote the string obtained from $w$ by removing every $1$. For example:

   \[
   \begin{align*}
   \text{obliviate}(\epsilon) &= \epsilon \\
   \text{obliviate}(00000) &= 00000 \\
   \text{obliviate}(11111) &= \epsilon \\
   \text{obliviate}(010001101) &= 00000
   \end{align*}
   \]

   Let $L$ be an arbitrary regular language.
   
   (a) **Prove** that the language $\{\text{obliviate}(w) \mid w \in L\}$ is regular.
   
   (b) **Prove** that the language $\{w \in \{0, 1\}^* \mid \text{obliviate}(w) \in L\}$ is regular.