Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check “Yes” if the statement is always true and “No” otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking “I don’t know” is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do not need to prove your answer is correct.

   Read each statement very carefully. Some of these are deliberately subtle.

   (a) If 2 + 2 = 5, then zero is odd.

   (b) Language L is regular if and only if there is a DFA that accepts every string in L.

   (c) Two languages L and L′ are regular if and only if L ∪ L′ is regular.

   (d) For every language L, if L∗ is empty, then L is empty.

   (e) Every regular language is recognized by a DFA with exactly one accepting state.

   (f) If L has a fooling set of size 374, then L is regular.

   (g) The language \( \{0^{374n} \mid n \geq 374\} \) is regular.

   (h) The language \( \{0^{37n}1^{4n} \mid n \geq 374\} \) is regular.

   (i) The language \( \{0^{3n}74n \mid n \leq 374\} \) is regular.

   (j) Every language is either regular or context-free.

2. For any string \( w \in \{0, 1\}^* \), let \( \text{slash}(w) \) be the string in \( \{0, 1, /\}^* \) obtained from \( w \) by inserting a new symbol / between any two consecutive appearances of the same symbol. For example:

\[
\text{slash}(\epsilon) = \epsilon \\
\text{slash}(10101) = 10101 \\
\text{slash}(001010111) = 0/010101/1/1
\]

For any language \( L \subseteq \{0, 1\}^* \), let \( \text{slash}(L) = \{\text{slash}(w) \mid w \in L\} \).

   (a) Draw or describe a DFA that accepts the language \( \text{slash}(\{0, 1\}^*) \).

   (b) Give a regular expression for the language \( \text{slash}(\{0, 1\}^*) \).

   (c) Prove that for any regular language \( L \), the language \( \text{slash}(L) \) is also regular.

(You do not need to justify your answers to parts (a) and (b).)
3. Let \( L \) be the language \( \{0^a1^b0^c \mid 2a = b + c\} \).

   (a) \textbf{Prove} that \( L \) is not a regular language.
   
   (b) Describe a context-free grammar for \( L \). (You do not need to justify your answer.)

4. For each of the following languages \( L \), give a regular expression that represents \( L \) and
draw or describe a DFA that recognizes \( L \). You do not need to justify your answers.

   (a) All strings in \( \{0, 1\}^* \) that do not contain either \( 100 \) or \( 011 \) as a substring
   
   (b) All strings in \( \{0, 1, 2\}^* \) that do not contain either \( 01 \) or \( 12 \) or \( 20 \) as a substring

5. For any string \( w \in \{0, 1\}^* \), let \( \text{stupefy}(w) \) denote the string obtained from \( w \) by deleting the
first 1 (if any) and replacing each remaining 1 with a 0. For example:

\[
\begin{align*}
\text{stupefy}(\varepsilon) &= \varepsilon \\
\text{stupefy}(000) &= 000 \\
\text{stupefy}(00100) &= 0000 \\
\text{stupefy}(111111) &= 000000 \\
\text{stupefy}(010001100) &= 000000000
\end{align*}
\]

Let \( L \) be an arbitrary regular language.

   (a) \textbf{Prove} that the language \( \{\text{stupefy}(w) \mid w \in L\} \) is regular.
   
   (b) \textbf{Prove} that the language \( \{w \in \{0, 1\}^* \mid \text{stupefy}(w) \in L\} \) is regular.