CS/ECE 374 A ✧ Fall 2019

Final Exam

December 16, 2019

Real name:	
NetID:	

Gradescope name:	
Gradescope email:	

- ***Don't panic!***
- If you brought anything except your writing implements and your two double-sided 8½" × 11" cheat sheets, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your real name, your university NetID, your Gradescope name, and your Gradescope email address in the boxes above. **We will not scan this page into Gradescope.**
- **Print the name you are using on Gradescope** at the top of every page of the answer booklet, except this cover page. These are the pages we will scan into Gradescope.
- Please do not write outside the black boxes on each page; these indicate the area of the page that the scanner can actually see.
- **Please read the entire exam before writing anything.** Please ask for clarification if any question is unclear.
- **The exam lasts 180 minutes.**
- If you run out of space for an answer, continue on the back of the page, or on the blank pages at the end of this booklet, **but please tell us where to look.** Alternatively, feel free to tear out the blank pages and use them as scratch paper.
- As usual, answering any (sub)problem with "I don't know" (and nothing else) is worth 25% partial credit. **Yes, even for problem 1.** Correct, complete, but suboptimal solutions are *always* worth more than 25%. A blank answer is not the same as "I don't know".
- **Please return your cheat sheets and all scratch paper with your answer booklet.**
- Good luck, and thanks for a great semester!

Beware of the man who works hard to learn something,
learns it, and finds himself no wiser than before.

He is full of murderous resentment of people who are ignorant
without having come by their ignorance the hard way.

— Bokonon

CS/ECE 374 A ✧ Fall 2019 **Final Exam Problem 1**	Gradescope name:

For each of the following questions, indicate ***every*** correct answer by marking the "Yes" box, and indicate ***every*** incorrect answer by marking the "No" box. **Assume P ≠ NP.** If there is any other ambiguity or uncertainty, mark the "No" box. For example:

Yes	~~No~~ ✗	IDK	$x + y = 5$
Yes	~~No~~ ✗	IDK	3SAT can be solved in polynomial time.
~~Yes~~ ✗	No	IDK	Jeff is not the Queen of England.
~~Yes~~ ✗	No	IDK	If $P = NP$ then Jeff is the Queen of England.

There are **40** yes/no choices altogether. Each correct choice is worth +½ point; each incorrect choice is worth −¼ point; each checked "IDK" is worth +1/8 point.

(a) Which of the following statements are true for ***at least one*** language $L \subseteq \{0, 1\}^*$?

Yes	No	IDK	L^* is empty.
Yes	No	IDK	L^* is not regular.
Yes	No	IDK	L^* is decidable.
Yes	No	IDK	L is decidable but L^* is undecidable.
Yes	No	IDK	L is the intersection of two regular languages, and L is undecidable.

(b) Which of the following statements are true for ***every*** language $L \subseteq \{0, 1\}^*$?

Yes	No	IDK	If L is not regular, then L is NP-hard.
Yes	No	IDK	If L is undecidable, then L is not regular.
Yes	No	IDK	If L is context-free, then L is infinite.
Yes	No	IDK	L is undecidable if and only if its reversal $L^R = \{w^R \mid w \in L\}$ is undecidable. (Recall that w^R denotes the reversal of the string w.)
Yes	No	IDK	L is undecidable if and only if its complement $\overline{L}$ is decidable.

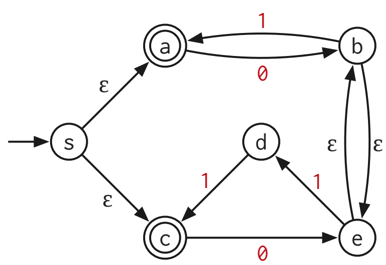
(c) Consider the following sets of undirected graphs:

- Trees is the set of all connected undirected graphs with no cycles.
- MostlyIndependent is the set of all undirected graphs that have an independent set containing at least half of the vertices. (Deciding whether a graph has this property is NP-hard.)

(For concreteness, assume that in both of these languages, graphs are represented by their adjacency matrices.) Which of the following ***must*** be true, assuming P≠NP?

Yes	No	IDK	Trees $\notin$ P
Yes	No	IDK	Trees $\subseteq$ MostlyIndependent
Yes	No	IDK	There is a polynomial-time reduction from Trees to MostlyIndependent
Yes	No	IDK	There is a polynomial-time reduction from MostlyIndependent to Trees
Yes	No	IDK	Trees is NP-hard.

(d) Let M be the following NFA:



Which of the following statements about M are true?

Yes	No	IDK	M rejects the empty string ε.
Yes	No	IDK	$\delta^*(s, \texttt{0101}) = \{a, d\}$
Yes	No	IDK	ε-reach$(s) = \{s, a, c\}$
Yes	No	IDK	M accepts the string 01101011
Yes	No	IDK	$L(M) = (\texttt{011})^* + (\texttt{01})^*$

(e) Which of the following languages over the alphabet $\Sigma = \{0, 1\}$ are ***regular***? Recall that $\#(a, w)$ denotes the number of times symbol a appears in string w.

Yes	No	IDK	$\{w \in \Sigma^* \mid \#(1, w) \text{ is a perfect square}\}$
Yes	No	IDK	The language generated by the context-free grammar $S \to 0S \mid S1 \mid \varepsilon$
Yes	No	IDK	$\{w \in \Sigma^* \mid \#(0, w) + \#(1, w) < 374\}$
Yes	No	IDK	$\{w \in \Sigma^* \mid \#(0, w) - \#(1, w) < 374\}$
Yes	No	IDK	The complement of a regular language

(f) Which of the following languages or problems are ***decidable***?

Yes	No	IDK	3Color
Yes	No	IDK	$\{\langle M \rangle \mid M$ accepts all non-empty strings but rejects the empty string $\varepsilon\}$
Yes	No	IDK	$\{\langle M \rangle \mid M$ accepts every string whose length is a perfect square$\}$
Yes	No	IDK	$\{\langle M \rangle \mid M$ is a Turing machine with at least two states$\}$
Yes	No	IDK	$\varnothing$

(g) Which of the following languages or problems can be proved undecidable ***using Rice's Theorem***?

Yes	No	IDK	3Color
Yes	No	IDK	$\{\langle M \rangle \mid M$ accepts all non-empty strings but rejects the empty string $\varepsilon\}$
Yes	No	IDK	$\{\langle M \rangle \mid M$ accepts every string whose length is a perfect square$\}$
Yes	No	IDK	$\{\langle M \rangle \mid M$ is a Turing machine with at least two states$\}$
Yes	No	IDK	$\varnothing$

(h) Suppose we want to prove that the following language is undecidable.

$$\text{MARVIN} := \left\{ \langle M \rangle \;\middle|\; M \text{ rejects an infinite number of strings} \right\}$$

Professor Beeblebrox, your instructor in Infinitely Improbable Galactic Presidencies, suggests a reduction from the standard halting language

$$\text{HALT} := \left\{ (\langle M \rangle, w) \;\middle|\; M \text{ halts on inputs } w \right\}.$$

Specifically, suppose there is a program PARANOIDANDROID that decides MARVIN. Professor Beeblebrox claims that the following algorithm decides HALT.

```
DECIDEHALT(⟨M⟩, w):
  Write code for the following algorithm:
      HEARTOFGOLD(x):
        run M on input w
        if x = VOGONPOETRY
            return FALSE
        else
            return TRUE
  return PARANOIDANDROID(⟨HEARTOFGOLD⟩)
```

Which of the following statements is true for all inputs $(\langle M \rangle, w)$?

Yes	No	IDK	If M accepts w, then HEARTOFGOLD accepts VOGONPOETRY.
Yes	No	IDK	If M rejects w, then HEARTOFGOLD rejects VOGONPOETRY.
Yes	No	IDK	If M hangs on w, then HEARTOFGOLD rejects EDDIE.
Yes	No	IDK	PARANOIDANDROID rejects ⟨HEARTOFGOLD⟩.
Yes	No	IDK	DECIDEHALT decides HALT; that is, Professor Beeblebrox's proof is correct.

CS/ECE 374 A ✧ Fall 2019 Final Exam Problem 2	Gradescope name:

You and your friends are planning a hiking trip in Jellystone National Park over winter break. You have a map of the park's trails that lists all the scenic views in the park but also warns that certain trail segments have a high risk of bear encounters. To make the hike worthwhile, you want to see at least three scenic views. You also don't want to get eaten by a bear, so you are willing to hike at most one high-bear-risk segment. Because the trails are narrow, each trail segment allows traffic in only one direction.

Your friend has converted the map into a directed graph $G = (V, E)$, where V is the set of intersections and E is the set of trail segments. A subset S of the edges are marked as *Scenic*; another subset B of the edges are marked as *high-Bear-risk*. You may assume that $S \cap B = \emptyset$. Each segment $e \in E$ is also labeled with a positive length $\ell(e)$ in miles. Your campsite appears on the map as a particular vertex $s \in V$, and the visitor center is another vertex $t \in V$.

Describe and analyze an algorithm to compute the shortest hike from your campsite s to the visitor center t that includes *at least* three scenic views and *at most* one high-bear-risk trail segment. You may assume such a hike exists.

CS/ECE 374 A ✧ Fall 2019 Final Exam Problem 3	Gradescope name:

For each of the following languages over the alphabet {0, 1}, state whether the language is regular or not, and then justify your answer as follows:

- If the language is regular, ***either*** give an regular expression that describes the language, ***or*** draw/describe a DFA or NFA that accepts the language. You do not need to prove that your automaton or regular expression is correct.
- If the language is not regular, ***prove*** that the language is not regular.

[Hint: Exactly one of these languages is regular.]

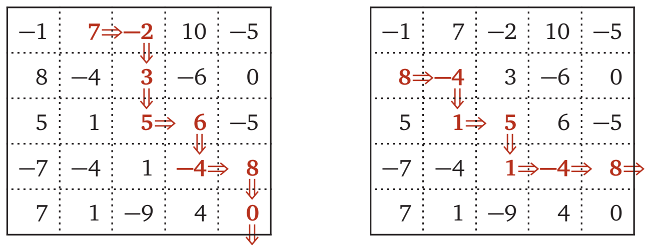
(a) $\{xy \mid x \text{ is a palindrome and } y \text{ is a palindrome}\}$

(b) $\{xy \mid x \text{ is a palindrome and } |x| \geq 2\}$

CS/ECE 374 A ✧ Fall 2019 **Final Exam Problem 4**	Gradescope name:

Vankin's Mile is an American solitaire game played on an $n \times n$ square grid. The player starts by placing a token on any square of the grid. Then on each turn, the player moves the token either one square to the right or one square down. The game ends when player moves the token off the edge of the board. Each square of the grid has a numerical value, which could be positive, negative, or zero. The player starts with a score of zero; whenever the token lands on a square, the player adds its value to his score. The object of the game is to score as many points as possible.

For example, given the grid shown below, the player can score $7-2+3+5+6-4+8+0=23$ points by following the path on the left, or they can score $8-4+1+5+1-4+8=15$ points by following the path on the right.



(a) Describe and analyze an efficient algorithm to compute the maximum possible score for a game of Vankin's Mile, given the $n \times n$ array of values as input.

(b) A variant called *Vankin's Niknav* adds an additional constraint to Vankin's Mile: *The sequence of values that the token touches must be a* ***palindrome***. Thus, the example path on the right is valid, but the example path on the left is not. Describe and analyze an efficient algorithm to compute the maximum possible score for an instance of Vankin's Niknav, given the $n \times n$ array of values as input.

4 (continued)

4 (continued)

CS/ECE 374 A ✧ Fall 2019 Final Exam Problem 5	Gradescope name:

Recall that a *satisfying assignment* for a 3CNF Boolean formula Φ assigns values (TRUE or FALSE) to the variables of Φ so that Φ evaluates to TRUE. A satisfying assignment is *balanced* if *exactly* half of the variables are set to TRUE.

The BALANCED3SAT problem asks whether a given 3CNF formula Φ has a balanced satisfying assignment. ***Prove*** that BALANCED3SAT is NP-hard.

CS/ECE 374 A ✧ Fall 2019 Final Exam Problem 6	Gradescope name:

Let M be an arbitrary NFA *without* ε-transitions, with input alphabet $\Sigma = \{0, 1\}$. Describe and analyze an efficient algorithm to decide whether M accepts an infinite number of strings.

(scratch paper)

(scratch paper)

(scratch paper)

Some useful NP-hard problems. You are welcome to use any of these in your own NP-hardness proofs, except of course for the specific problem you are trying to prove NP-hard.

CircuitSat: Given a boolean circuit, are there any input values that make the circuit output True?

3Sat: Given a boolean formula in conjunctive normal form, with exactly three distinct literals per clause, does the formula have a satisfying assignment?

MaxIndependentSet: Given an undirected graph G, what is the size of the largest subset of vertices in G that have no edges among them?

MaxClique: Given an undirected graph G, what is the size of the largest complete subgraph of G?

MinVertexCover: Given an undirected graph G, what is the size of the smallest subset of vertices that touch every edge in G?

MinSetCover: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S, what is the size of the smallest subcollection whose union is S?

MinHittingSet: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S, what is the size of the smallest subset of S that intersects every subset S_i?

3Color: Given an undirected graph G, can its vertices be colored with three colors, so that every edge touches vertices with two different colors?

HamiltonianPath: Given graph G (either directed or undirected), is there a path in G that visits every vertex exactly once?

HamiltonianCycle: Given a graph G (either directed or undirected), is there a cycle in G that visits every vertex exactly once?

TravelingSalesman: Given a graph G (either directed or undirected) with weighted edges, what is the minimum total weight of any Hamiltonian path/cycle in G?

LongestPath: Given a graph G (either directed or undirected, possibly with weighted edges), what is the length of the longest simple path in G?

SteinerTree: Given an undirected graph G with some of the vertices marked, what is the minimum number of edges in a subtree of G that contains every marked vertex?

SubsetSum: Given a set X of positive integers and an integer k, does X have a subset whose elements sum to k?

Partition: Given a set X of positive integers, can X be partitioned into two subsets with the same sum?

3Partition: Given a set X of $3n$ positive integers, can X be partitioned into n three-element subsets, all with the same sum?

IntegerLinearProgramming: Given a matrix $A \in \mathbb{Z}^{n \times d}$ and two vectors $b \in \mathbb{Z}^n$ and $c \in Z^d$, compute $\max\{c \cdot x \mid Ax \leq b, x \geq 0, x \in \mathbb{Z}^d\}$.

FeasibleILP: Given a matrix $A \in \mathbb{Z}^{n \times d}$ and a vector $b \in \mathbb{Z}^n$, determine whether the set of feasible integer points $\max\{x \in \mathbb{Z}^d \mid Ax \leq b, x \geq 0\}$ is empty.

Draughts: Given an $n \times n$ international draughts configuration, what is the largest number of pieces that can (and therefore must) be captured in a single move?

SuperMarioBrothers: Given an $n \times n$ Super Mario Brothers level, can Mario reach the castle?

SteamedHams: Aurora borealis? At this time of year, at this time of day, in this part of the country, localized entirely within your kitchen? May I see it?