Theorem 0.1. Rice’s Theorem Let $L$ be any set of languages that satisfies the following conditions:
- There is a Turing machine $Y$ such that $\text{Accept}(Y) \in L$.
- There is a Turing machine $N$ such that $\text{Accept}(N) \notin L$.

The language $\text{AcceptIn}(L) := \{ \langle M \rangle \mid \text{Accept}(M) \in L \}$ is undecidable.

Prove that the following languages are undecidable using Rice’s Theorem:

1. $\text{AcceptRegular} := \{ \langle M \rangle \mid \text{Accept}(M) \text{ is regular} \}$
2. $\text{AcceptIllini} := \{ \langle M \rangle \mid M \text{ accepts the string } \text{ILLINI} \}$
3. $\text{AcceptPalindrome} := \{ \langle M \rangle \mid M \text{ accepts at least one palindrome} \}$
4. $\text{AcceptThree} := \{ \langle M \rangle \mid M \text{ accepts exactly three strings} \}$
5. $\text{AcceptUndecidable} := \{ \langle M \rangle \mid \text{Accept}(M) \text{ is undecidable} \}$

To think about later. Which of the following are undecidable? How would you prove that?

1. $\text{Accept}\{\varepsilon\} := \{ \langle M \rangle \mid M \text{ accepts only the string } \varepsilon; \text{ that is, } \text{Accept}(M) = \{\varepsilon\} \}$
2. $\text{Accept}\emptyset := \{ \langle M \rangle \mid M \text{ does not accept any strings; that is, } \text{Accept}(M) = \emptyset \}$
3. $\text{Accept}\not\emptyset := \{ \langle M \rangle \mid \text{Accept}(M) \text{ is not an acceptable language} \}$
4. $\text{Accept}=\text{Reject} := \{ \langle M \rangle \mid \text{Accept}(M) = \text{Reject}(M) \}$
5. $\text{Accept} \neq \text{Reject} := \{ \langle M \rangle \mid \text{Accept}(M) \neq \text{Reject}(M) \}$
6. $\text{Accept}\cup\text{Reject} := \{ \langle M \rangle \mid \text{Accept}(M) \cup \text{Reject}(M) = \Sigma^* \}$