Prove that each of the following languages is *not* regular.

1. $\{0^{2^n} \mid n \geq 0\}$.
2. $\{0^{2n}1^n \mid n \geq 0\}$.
3. $\{0^m1^n \mid m \neq 2n\}$.
4. Strings over $\{0, 1\}$ where the number of 0s is exactly twice the number of 1s.
5. Strings of properly nested parentheses $()$, brackets $[ ]$, and braces $\{ \}$. For example, the string $([)]\{\}$ is in this language, but the string $([])$ is not, because the left and right delimiters don’t match.
6. Strings of the form $w_1\#w_2\#\cdots\#w_n$ for some $n \geq 2$, where each substring $w_i$ is a string in $\{0, 1\}^*$, and some pair of substrings $w_i$ and $w_j$ are equal.

**Extra problems**

7. $\{0^{n^2} \mid n \geq 0\}$.
8. $\{w \in (0 + 1)^* \mid w \text{ is the binary representation of a perfect square}\}$.