Designing DFAs via product construction and designing NFAs.

1. Describe a DFA that accepts the following language over the alphabet \( \Sigma = \{0, 1\} \).
   All strings in which the number of 0s is even and the number of 1s is not divisible by 3.
2. All strings that are both the binary representation of an integer divisible by 3 and the ternary (base-3) representation of an integer divisible by 4.
   For example, the string 1100 is an element of this language, because it represents \(2^3 + 2^2 = 12\) in binary and \(3^3 + 3^2 = 36\) in ternary.
3. Design an NFA for the language \((01)^+ + (010)^+\).

Work on these later:
Describe deterministic finite-state automata that accept each of the following languages over the alphabet \( \Sigma = \{0, 1\} \). You may find it easier to describe these DFAs formally than to draw pictures.

4. All strings \(w\) such that \(\left\lfloor \frac{|w|}{2} \right\rfloor \mod 6 = 4\). (Hint: Maintain both \(\left\lfloor \frac{|w|}{2} \right\rfloor \mod 6\) and \(|w| \mod 6\).)

5. (Hard.) All strings \(w\) such that \(F_{\#(10,w)} \mod 10 = 4\), where \(\#(10,w)\) denotes the number of times \(10\) appears as a substring of \(w\), and \(F_n\) is the \(n\)th Fibonacci number:

\[
F_n = \begin{cases} 
0 & \text{if } n = 0 \\
1 & \text{if } n = 1 \\
F_{n-1} + F_{n-2} & \text{otherwise}
\end{cases}
\]