Give regular expressions for each of the following languages over the alphabet \{0, 1\}.

1. All strings containing the substring 000.
2. All strings not containing the substring 000.
3. All strings in which every run of 0s has length at least 3.
4. All strings in which 1 does not appear after a substring 000.
5. All strings containing at least three 0s.
6. Every string except 000. (Hint: Don’t try to be clever.)

Work on these later:

7. All strings \(w\) such that in every prefix of \(w\), the number of 0s and 1s differ by at most 1. Solution: Equivalently, strings that alternate between 0s and 1s: \((01 + 10)^* ( + 0 + 1)\)

8. (Hard.) All strings containing at least two 0s and at least one 1.

Solution: There are three possibilities for how such a string can begin:

- Start with 00, then any number of 0s, then 1, then anything.
- Start with 01, then any number of 1s, then 0, then anything.
- Start with 1, then a substring with exactly two 0s, then anything.

All together: \(000^* 1(0 + 1)^* + 011^* 0(0 + 1)^* + 11^* 01^* 0(0 + 1)^*\)

9. (Hard.) All strings \(w\) such that in every prefix of \(w\), the number of 0s and 1s differ by at most 2.

10. (Really hard.) All strings in which the substring 000 appears an even number of times.

(For example, 0001000 and 0000 are in this language, but 00000 is not.)