Give regular expressions for each of the following languages over the alphabet \{0, 1\}.

1. All strings containing the substring \textbf{000}.
2. All strings \textit{not} containing the substring \textbf{000}.
3. All strings in which every run of 0s has length at least 3.
4. All strings in which 1 does not appear after a substring \textbf{000}.
5. All strings containing at least three 0s.
6. Every string except \textbf{000}. (\textbf{Hint}: Don't try to be clever.)

\textbf{Work on these later:}

7. All strings \(w\) such that \textit{in every prefix of }\(w\), the number of 0s and 1s differ by at most 1. \textit{Solution:} Equivalently, strings that alternate between 0s and 1s: \((01 + 10)^* (0 + 1)\)

8. (\textbf{Hard.}) All strings containing at least two 0s and at least one 1. \textit{Solution:} There are three possibilities for how such a string can begin:
   
   \begin{itemize}
   \item Start with \textbf{00}, then any number of 0s, then 1, then anything.
   \item Start with \textbf{01}, then any number of 1s, then 0, then anything.
   \item Start with 1, then a substring with exactly two 0s, then anything.
   \end{itemize}

   All together: \(00^*1(0 + 1)^* + 011^*0(0 + 1)^* + 11^*01^*0(0 + 1)^*\)

9. (\textbf{Hard.}) All strings \(w\) such that \textit{in every prefix of }\(w\), the number of 0s and 1s differ by at most 2.

10. (\textbf{Really hard.}) All strings in which the substring \textbf{000} appears an even number of times. 
    (For example, \textbf{0001000} and \textbf{0000} are in this language, but \textbf{00000} is not.)