In lecture, Alex described an algorithm of Karatsuba that multiplies two $n$-digit integers using $O(n^{\lg 3})$ single-digit additions, subtractions, and multiplications. In this lab we'll look at some extensions and applications of this algorithm.

1. Describe an algorithm to compute the product of an $n$-digit number and an $m$-digit number, where $m < n$, in $O(m^{\lg 3} - 1)n$ time.

2. Describe an algorithm to compute the decimal representation of $2^n$ in $O(n^{\lg 3})$ time. (The standard algorithm that computes one digit at a time requires $\Theta(n^2)$ time.)

3. Describe a divide-and-conquer algorithm to compute the decimal representation of an arbitrary $n$-bit binary number in $O(n^{\lg 3})$ time. [Hint: Let $x = a \cdot 2^{n/2} + b$. Watch out for an extra log factor in the running time.]

Think about later:

4. Suppose we can multiply two $n$-digit numbers in $O(M(n))$ time. Describe an algorithm to compute the decimal representation of an arbitrary $n$-bit binary number in $O(M(n) \log n)$ time.