Recall that $w^R$ denotes the reversal of string $w$; for example, $TURING^R = GNIRUT$. Prove that the following language is undecidable.

\[ \text{RevAccept} := \{ \langle M \rangle \mid M \text{ accepts } \langle M \rangle^R \} \]

Note that Rice’s theorem does not apply to this language.

Let $M$ be a Turing machine, let $w$ be an arbitrary input string, and let $s$ be an integer. We say that $M$ accepts $w$ in space $s$ if, given $w$ as input, $M$ accesses only the first $s$ (or fewer) cells on its tape and eventually accepts.

2.A. Sketch a Turing machine/algorithm that correctly decides the following language:

\[ \{ \langle M, w \rangle \mid M \text{ accepts } w \text{ in space } |w|^2 \} \]

2.B. Prove that the following language is undecidable:

\[ \{ \langle M \rangle \mid M \text{ accepts at least one string } w \text{ in space } |w|^2 \} \]

Consider the language SometimesHalt = \{ \langle M \rangle \mid M \text{ halts on at least one input string} \}. Note that $\langle M \rangle \in$ SometimesHalt does not imply that $M$ accepts any strings; it is enough that $M$ halts on (and possibly rejects) some string.

3.A. Prove that SometimesHalt is undecidable.

3.B. Sketch a Turing machine/algorithm that accepts SometimesHalt.

For each of the following languages, either prove that the language is decidable, or prove that the language is undecidable.

4.A. $L_0 = \{ \langle M \rangle \mid \text{given any input string, } M \text{ eventually leaves its start state} \}

4.B. $L_1 = \{ \langle M \rangle \mid M \text{ decides } L_0 \}

4.C. $L_2 = \{ \langle M \rangle \mid M \text{ decides } L_1 \}

4.D. $L_3 = \{ \langle M \rangle \mid M \text{ decides } L_2 \}

4.E. $L_4 = \{ \langle M \rangle \mid M \text{ decides } L_3 \}

Rubric: 10 points: 4 for part (a) + 3/2 for each other part.

Rubric: [for all undecidability proofs, out of 10 points]

Diagonalization:

1. 4 for correct wrapper Turing machine
2. 6 for self-contradiction proof (= 3 for $\Leftarrow$ + 3 for $\Rightarrow$)

Reduction:

1. 4 for correct reduction
2. 3 for “if” proof
3. 3 for “only if” proof

Rice’s Theorem:

1. 4 for positive Turing machine
2. 4 for negative Turing machine
3. 2 for other details (including using the correct variant of Rice’s Theorem)