HW 1

Due on **Wednesday, September 13, 2017 at 10am**

CS/ECE 374: Algorithms & Models of Computation, Fall 2017 Version: **1.22**

For waitlist information for the course, please see here.

Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.

1 (100 PTS.) Regularize this.

For each of the following languages over the alphabet $\{0, 1\}$, give a regular expression that describes that language, and briefly argue why your expression is correct.

1.A. All strings except 010 and 101.

1.B. All strings that starts in 10 and contain 111 as a substring.

1.C. All strings in which every nonempty maximal substring of consecutive 1s is of even length. For instance 10011 is not in the language while 01111000110 is.

1.D. All strings that do not contain the substring 010.

1.E. All strings that do not contain the subsequence 101.

2 (100 PTS.) Then, shalt thou count to three.

Let L be the set of all strings in $\{0, 1\}^*$ that contain at least three occurrences of the substring 001.

2.A. Describe a DFA that over the alphabet $\Sigma = \{0, 1\}$ that accepts the language L. Argue that your machine accepts every string in L and nothing else, by explaining what each state in your DFA *means*.

You may either draw the DFA or describe it formally, but the states Q, the start state s, the accepting states A, and the transition function δ must be clearly specified.

2.B. Give a regular expression for L, and briefly argue why the expression is correct.

3 (100 PTS.) Do not affix it.

(This exercise is about writing things formally – it is not difficult once you have cut through the formalism. In short, don't panic - you can do it!)

3.A. Let $M = (Q, \Sigma, \delta, s, A)$ be a DFA. A state $q \in Q$ is ***bad***, if for all strings $w \in \Sigma^*$ we have that $\delta^*(q, w) \notin A$. Let $B(M) \subseteq Q$ be the set of bad states of M. Consider the DFA $M' = (Q, \Sigma, \delta, s, B(M))$. What is the language $L(M')$? Prove formally your answer!

3.B. Prove that if $x \in L(M')$ and $y \in \Sigma^*$, then $xy \in L(M')$.

3.C. Let L_1 and L_2 be two regular languages over Σ accepted by DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$, and $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$, respectively.

Describe a DFA $M = (Q, \Sigma, \delta, s, A)$ in terms of M_1 and M_2 that accepts

$$L = \{w \mid w \in L_2 \text{ and no prefix of } w \text{ is in } L_1\}$$

Formally specify the components Q, δ, s, and A for M in terms of the components of M_1 and M_2.