1. **Don’t panic!**
2. Please print your name and NetID in each page in the appropriate fields, and circle your discussion section in the boxes above. We will return your exam at the indicated section.
3. If you brought anything except your writing implements, your double-sided handwritten (in the original) 8½” × 11” cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, you please turn off and put away all medically unnecessary electronic devices.
   - Submit your cheat sheet together with your exam. An exam without your cheat sheet attached to it will not be graded.
   - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
4. Please ask for clarification if any question is unclear.
5. This exam lasts 120 minutes. The clock started when you got the questions.
6. If you run out of space for an answer, feel free to use the blank pages at the back of this booklet, but please tell us where to look.
7. As usual, answering any (sub)problem with I don’t know (and nothing else) is worth 25% partial credit. Correct, complete, but sub-optimal solutions are always worth more than 25%. A blank answer is not the same as I don’t know.
8. Total IDK points for the whole exam would not exceed 10.
9. Beware the Three Deadly Sins. Give complete solutions, not examples. Declare all your variables. If you don’t know the answer admit it and use IDK.
10. Style counts. Please use the backs of the pages or the blank pages at the end for scratch work, so that your actual answers are clear.
11. Please return all paper with your answer booklet: your cheat sheet, and all scratch paper. We will return everything with your graded exam.
12. **Good luck!**
For each statement below, check “True” if the statement is **always** true and “False” otherwise. Each correct answer is worth 2 points; each incorrect answer is worth 0 points; and flipping a coin is (on average) worth 1 point. **There is no IDK for this question.**

1. **A.** Consider the logical statement “If the moon is made of silver, then the sun is made of chicken.” This expression is:  

   - False: [ ]  
   - True: [ ]

2. **B.** Let $L$ be a regular language over alphabet $\Sigma$, and consider the language  

   $$L' = \{xy \mid x, y \in \Sigma^*, \alpha \in \Sigma, \text{ and } x\alpha y \in L\}.$$  

   The language $L'$ is regular.

   - False: [ ]  
   - True: [ ]

3. **C.** For all context-free languages $L$ and $L'$, the language $(L \cdot L') \cup (L' \cdot L)$ is also context-free.

   - False: [ ]  
   - True: [ ]

4. **D.** If a language $L \subseteq \{0, 1\}^*$ contains a string of length one, then $L^*$ is regular.

   - False: [ ]  
   - True: [ ]

5. **E.** If $L_1, L_2, \ldots$ are all regular languages, then $L = \bigcup_{i=0}^{\infty} L_i$ is regular.

   - False: [ ]  
   - True: [ ]

6. **F.** For all languages $L$, if $L$ is regular, then $L$ does not have an infinite fooling set.

   - False: [ ]  
   - True: [ ]

7. **G.** For all languages $L, L' \subseteq \Sigma^*$, if $L$ and $L'$ are recognized by DFAs $M$ and $M'$, respectively, then $L' \setminus L$ can be represented by a regular expression.

   - False: [ ]  
   - True: [ ]

8. **H.** $\{0^i1^j0^k1^\ell \mid i, j, k, \ell \geq 0\}$ is not regular.

   - False: [ ]  
   - True: [ ]

9. **I.** Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary NFAs with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \setminus L(M') = L(M)$.

   - False: [ ]  
   - True: [ ]

10. **J.** The strings 010 and 101 are distinguishable by the language $L = \{x \in \Sigma^* \mid |x| \text{ is even}\}$.

   - False: [ ]  
   - True: [ ]
For each of the following languages over the alphabet \( \Sigma = \{0, 1\} \), either prove that the language is regular or prove that the language is not regular. **Exactly one of these two languages is regular.** [This is a tricky question.]

2.A. \( L = \{1^n w 0^{m-n} \mid w \in \Sigma^+ \text{ and } m \geq n > 0\} \).

2.B. \( L = \{x0^n x^R \mid x \in \Sigma^+ \text{ and } n > 0\} \), where \( x^R \) is the reverse string of \( x \).
For any language $L$, let $\text{mid}(L) = \{ y \mid xyz \in L \text{ for some } x, y, z \in \Sigma^* \}$ be the language containing all substrings of all strings in $L$. For example, if $L = \{000, 100, 110, 111\}$, then

$$\text{mid}(L) = \{\varepsilon, 0, 00, 000, 1, 10, 100, 11, 110, 111\}.$$ 

**Prove** that for any regular language $L$, the language $\text{mid}(L)$ is also regular (suggestion: first describe the necessary construction, and then prove the correctness of the construction).
In the following, you do not need to prove that your answers are correct.

4.A. Provide a DFA and a regular expression for the following language: The set of all strings in \(\{a, b\}\)^* that do not contain the substring \(aaaaa\).

4.B. Provide a regular expression for the following language: The set of all strings in \(\{a, b\}\)^* that contain both \(ab\) and \(aa\) as substrings.
A CFG $G$ is *silly* if all production rules are of the form $A \to cB$ or $A \to \varepsilon$, where $c \in \Sigma$ and $A$ and $B$ are variables (i.e., non-terminals) of $G$. For a given silly CFG grammar $G$, provide a construction that shows that the language of $L(G)$ is regular. Provide a convincing argument why this is true (you do not need to provide a formal proof).

[If you do not know the answer - just use IDK – no need to waste your and our time.]