Deterministic Finite Automata

Lecture 4

Input Accepted by a DFA

We say that M **accepts** $w \in \Sigma^*$ if M, on input w, starting from the start state s, reaches a final state

i.e., $\delta^*(s,w) \in F$

$L(M)$ is the set of all strings accepted by M

i.e., $L(M) = \{ w \mid \delta^*(s,w) \in F \}$

Called the language accepted by M

Input Accepted by a DFA

What kind of language is accepted by FSM?

- Automatic (it is an automaton after all)!

- We will use: REGULAR (not a coincidence)

Language is regular iff

-it is accepted by a finite state automaton

-it is described by a regular expression

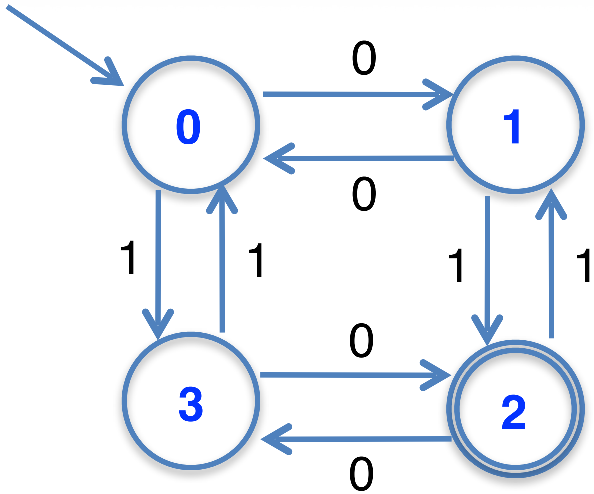
Warning

"M accepts language L" does not mean simply that M accepts each string in L.

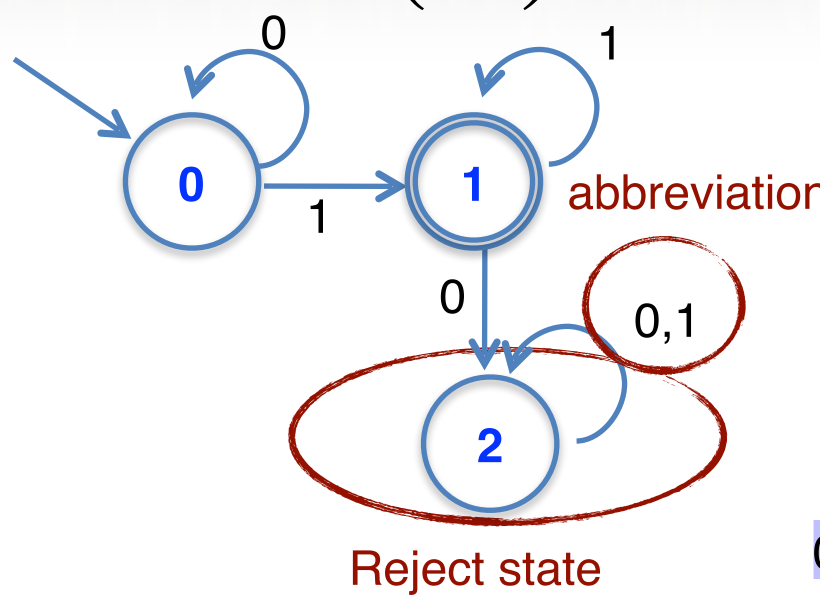
"M accepts language L" means M accepts each string in L and no others!

$$L(M) = L$$

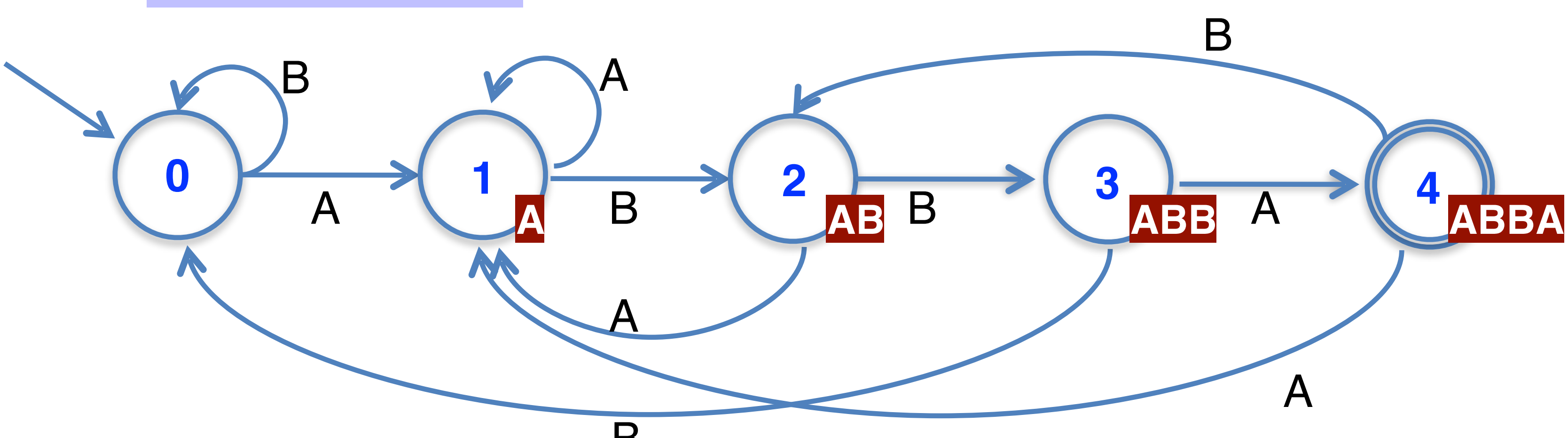
Examples: What is $L(M)$?



odd #0 and odd #1



0*11*



(A+B)*ABBA

Building DFAs

State = Memory

First, decide on Q

The state of a DFA is its entire memory of what has come before

The state must capture enough information to complete the computation on the suffix to come

When designing a DFA, think “what do I need to know at this moment?” That is your state.

DFA Construction Exercise

$L(M) = \{w \mid w$ contains 00 $\}$

Is it regular??

(0+1)*00(0+1)*

What should be in the memory?

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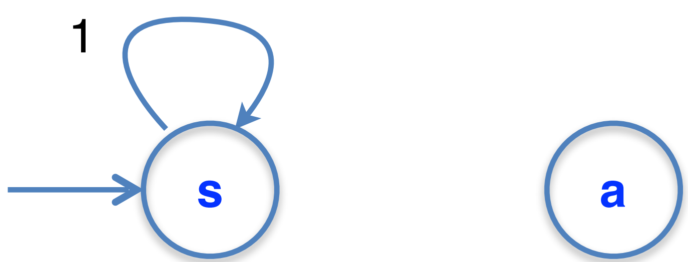

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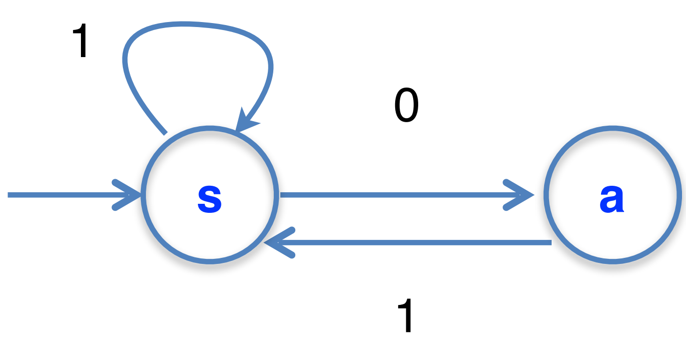

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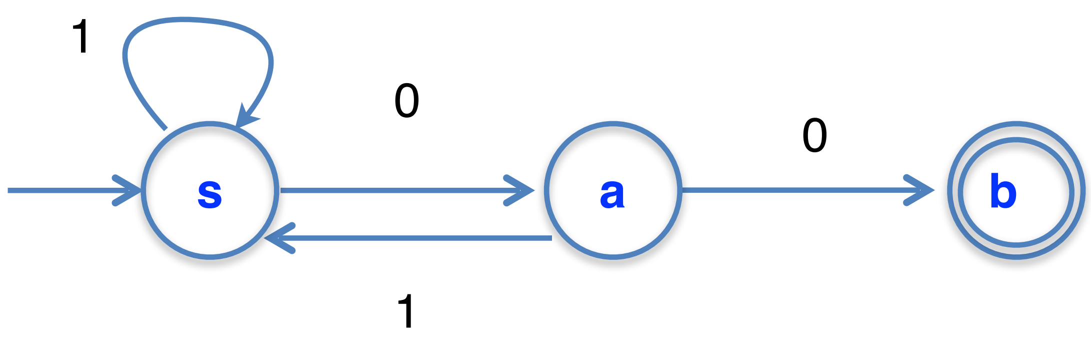

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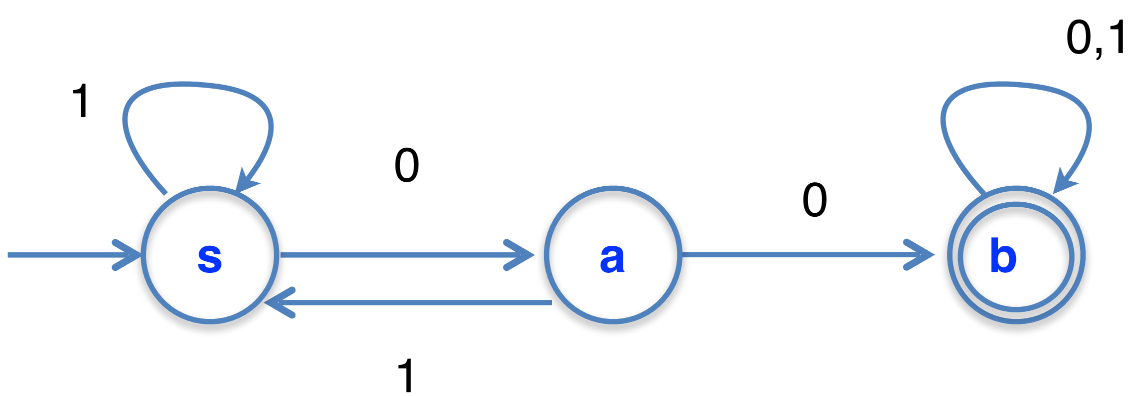


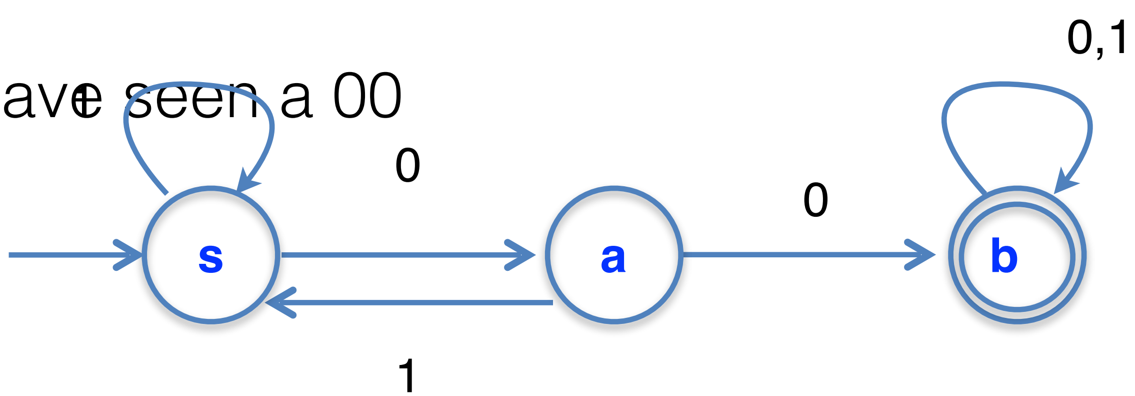
DFA Construction Exercise

$L(M) = \{w \mid w$ contains 00 $\}$

- s : I haven't seen a 00, previous symbol was 1 or undefined.

- a: I haven't seen a 00, previous symbol was a 0

- b: I have seen a 00



- We have exhausted of all strings. Either accepted (with 00) or not.

DFA construction

- Make sure you interpret all the cases!
- How about design a DFA for $L(M) = \{w \mid w$ contains 001100110011111001101101$\}$?
- There is algorithm to minimize the DFA, but when you are asked to do it, try to be clear versus succinct.
- Try to be "stupid", do brute force!!!
- When you are just trying to prove that a language is regular —> DFA for the language exists. Write an algorithm like we did for multiple of 5!

A More Complicated example

$L(M) = \{w \mid w$ contains 00 and then 11$\}$

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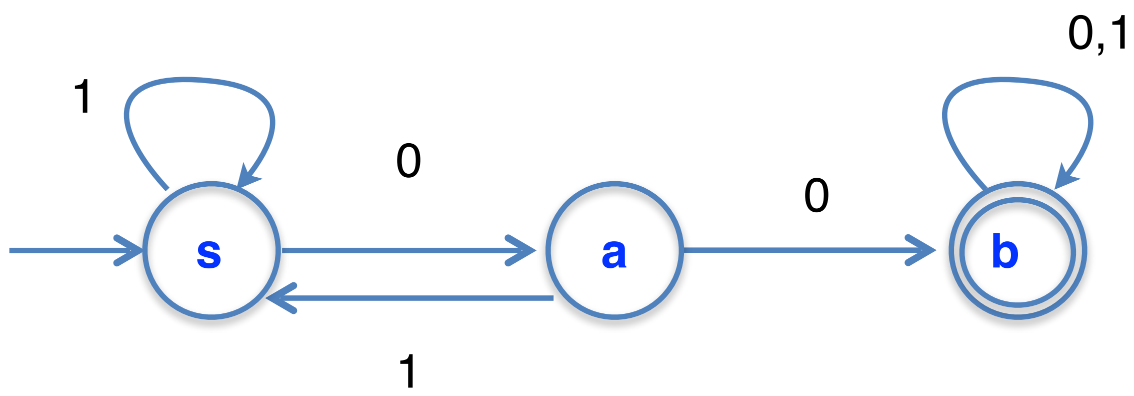

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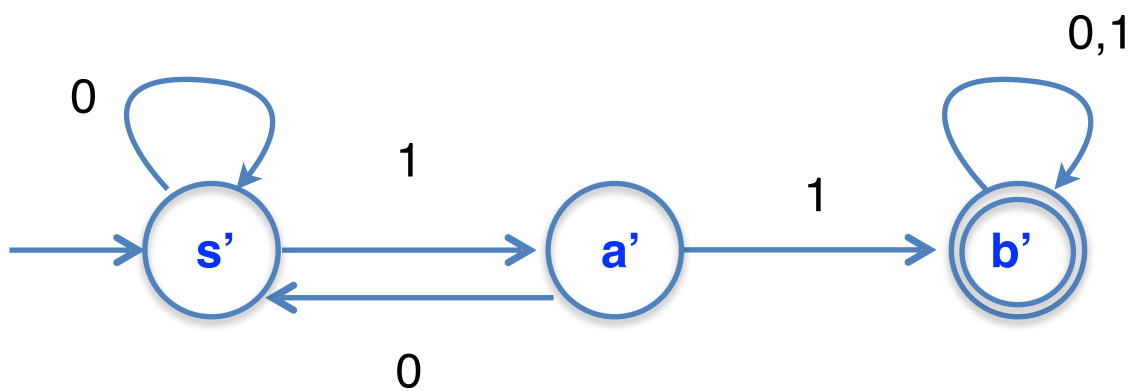

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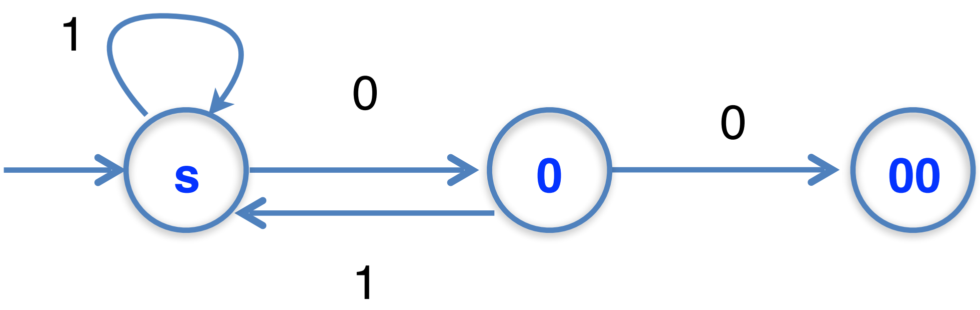

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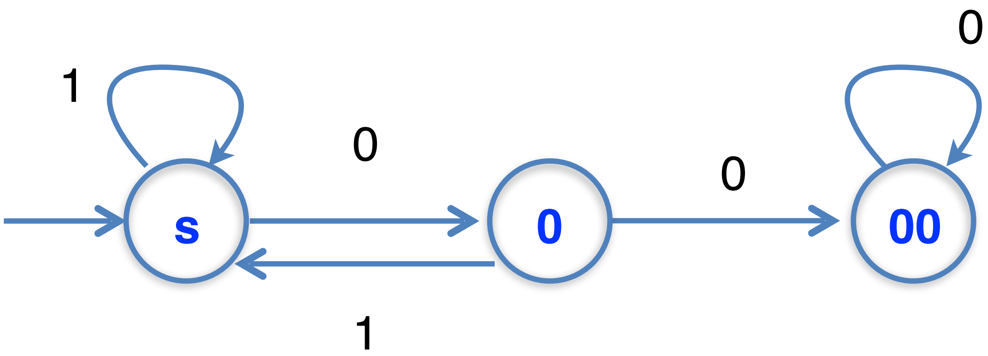

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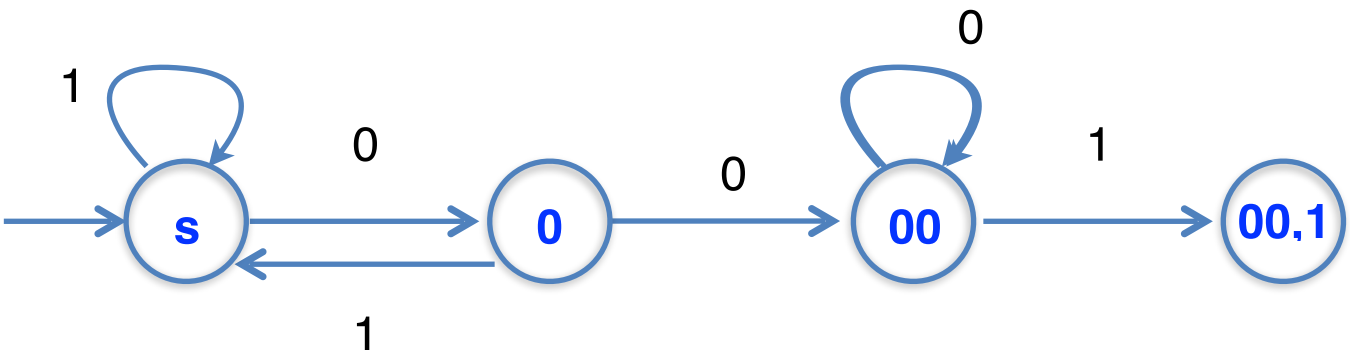

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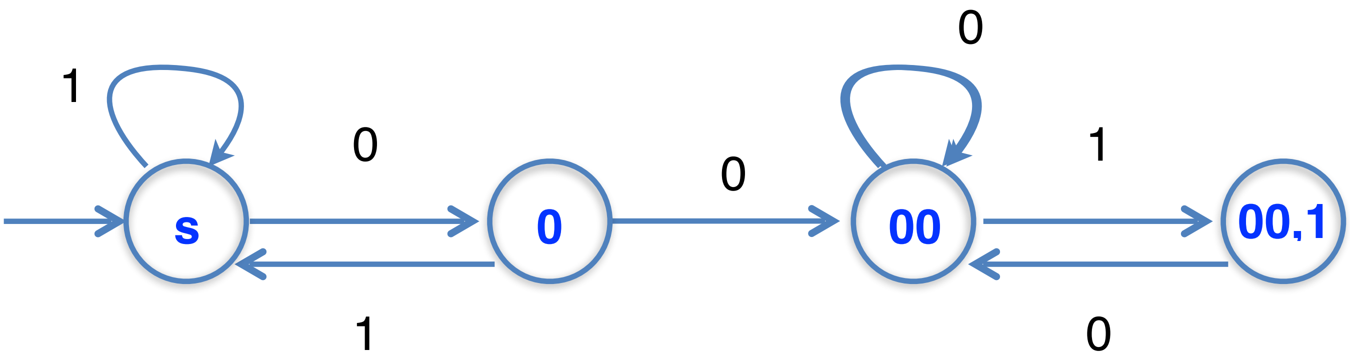

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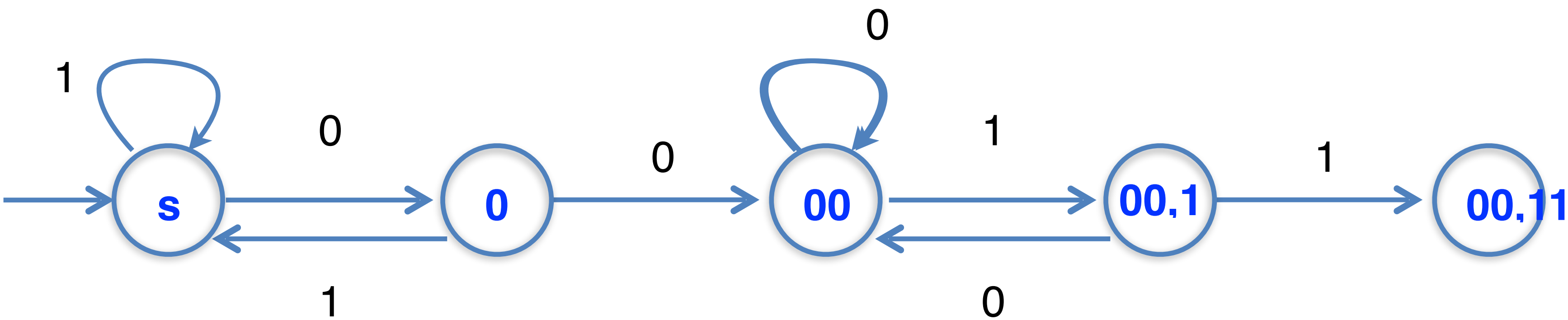

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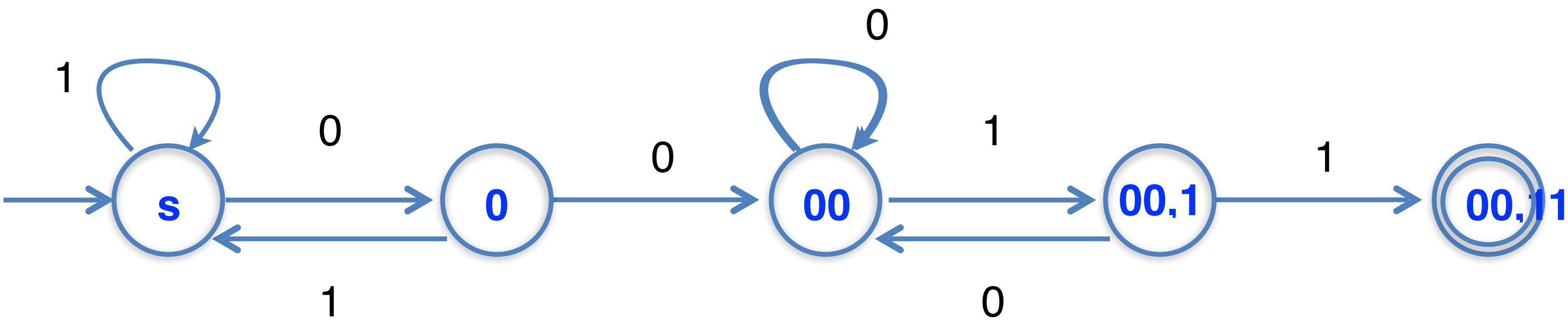

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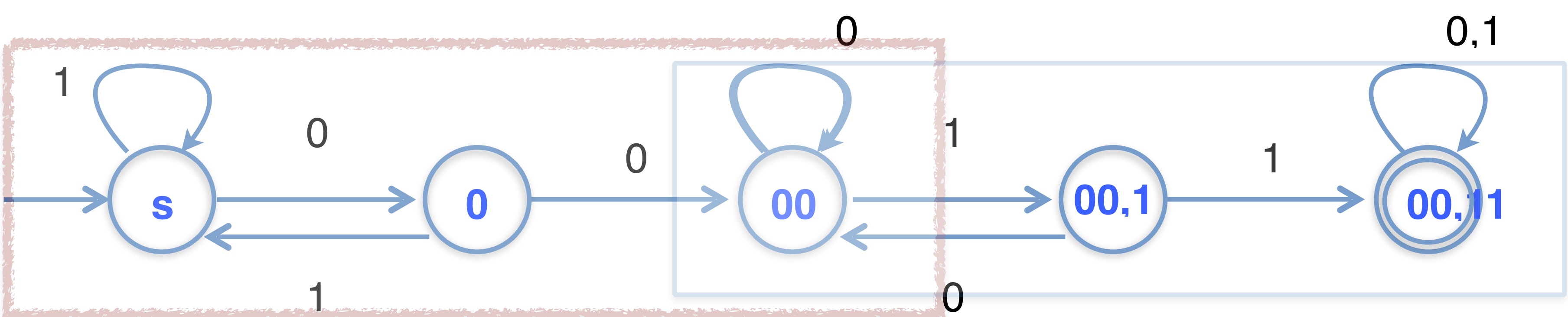

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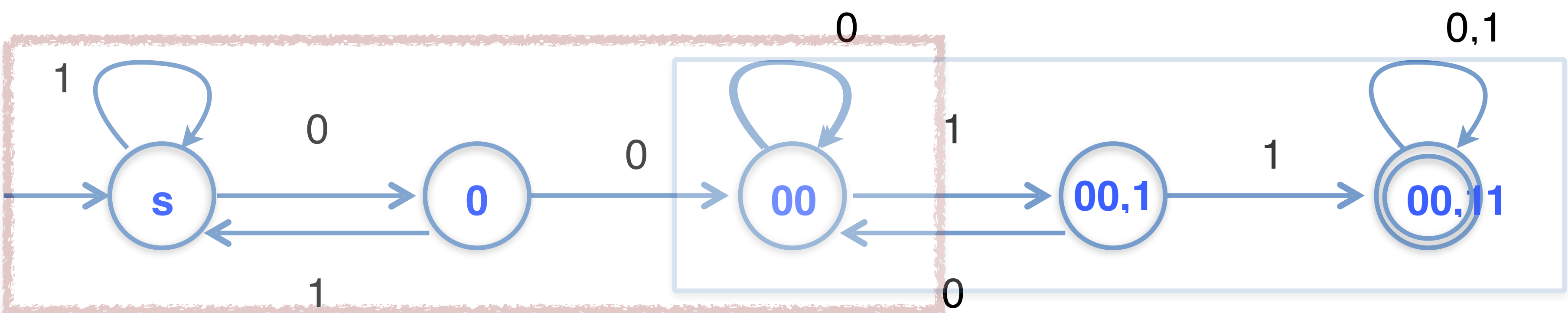

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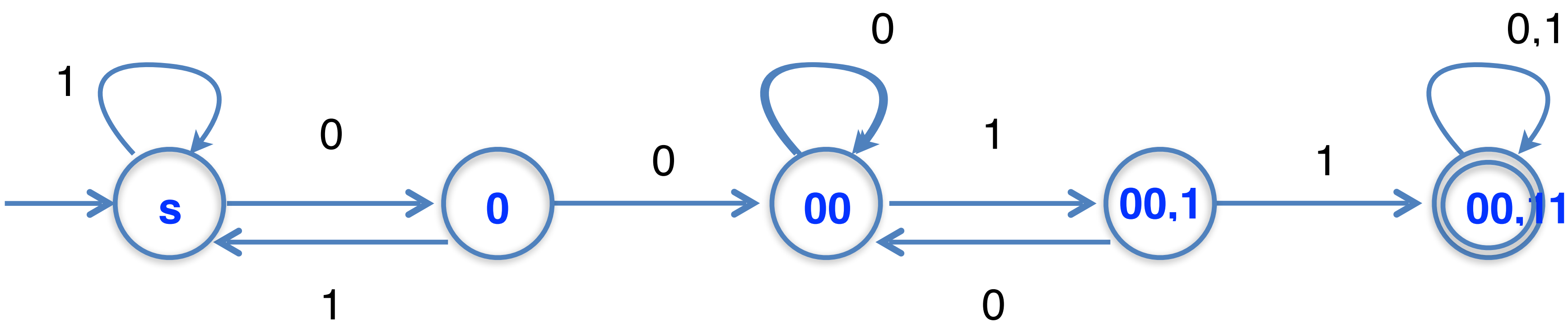
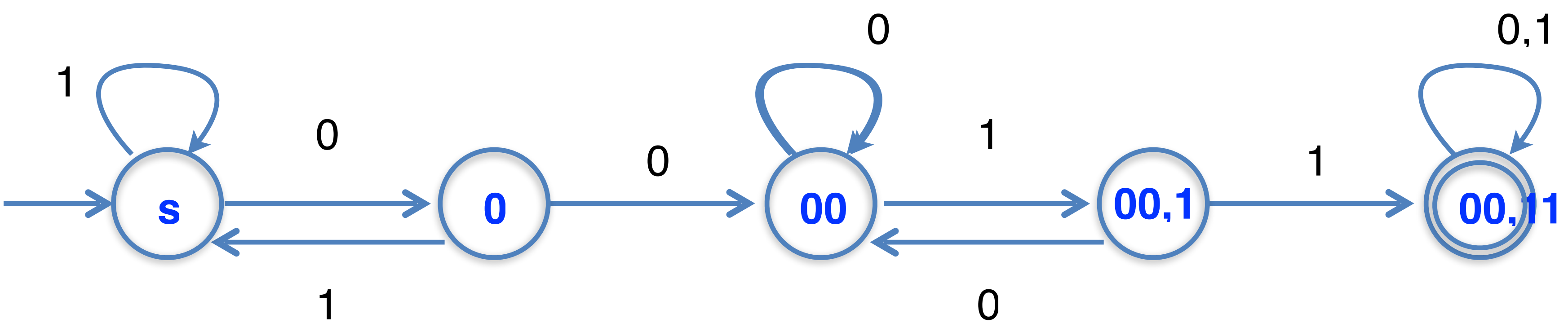


- If A and B are regular, then AB is regular. Does the same hold for DFA?
- NO! you cannot glue two DFAs together in general like that. This was a special case

What about the complement?

$L(M) = \{w \mid w$ contains no 11 after 00$\}$



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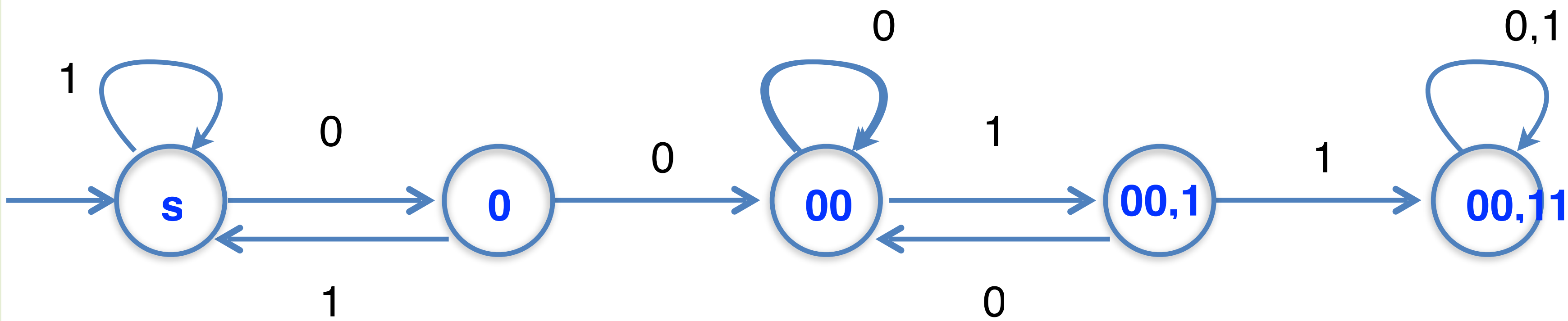

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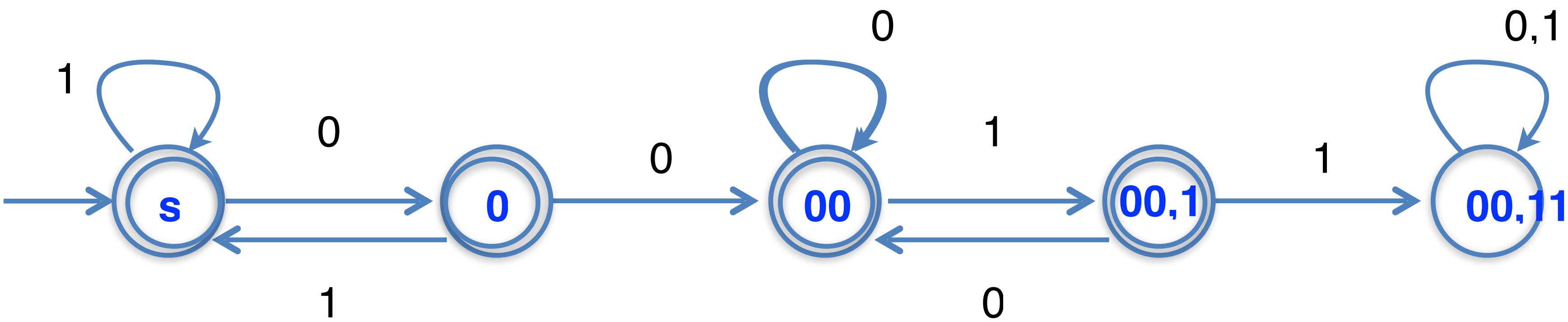
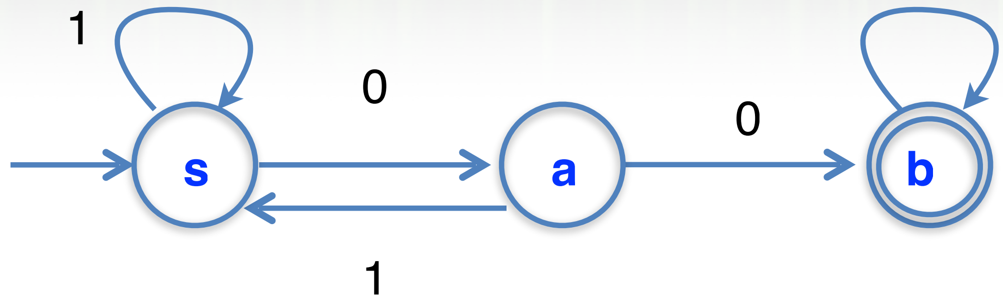
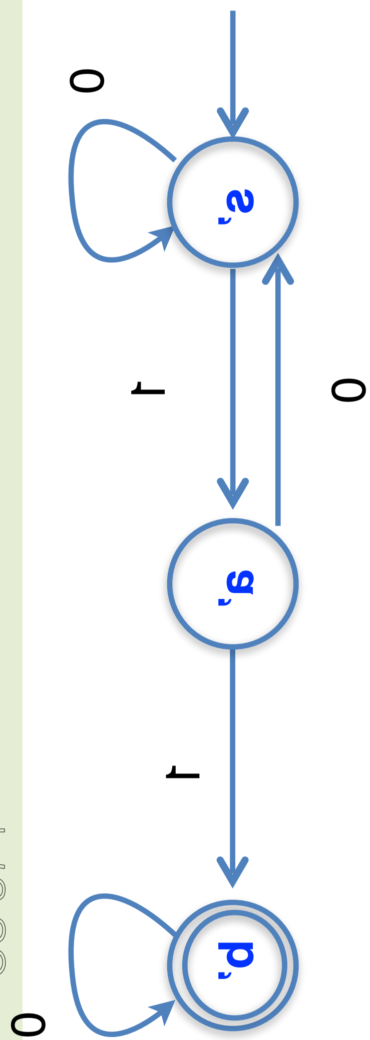


- If L is regular, then Σ*\L is regular

- Make the accepting states into non-accepting and the non-accepting states into accepting

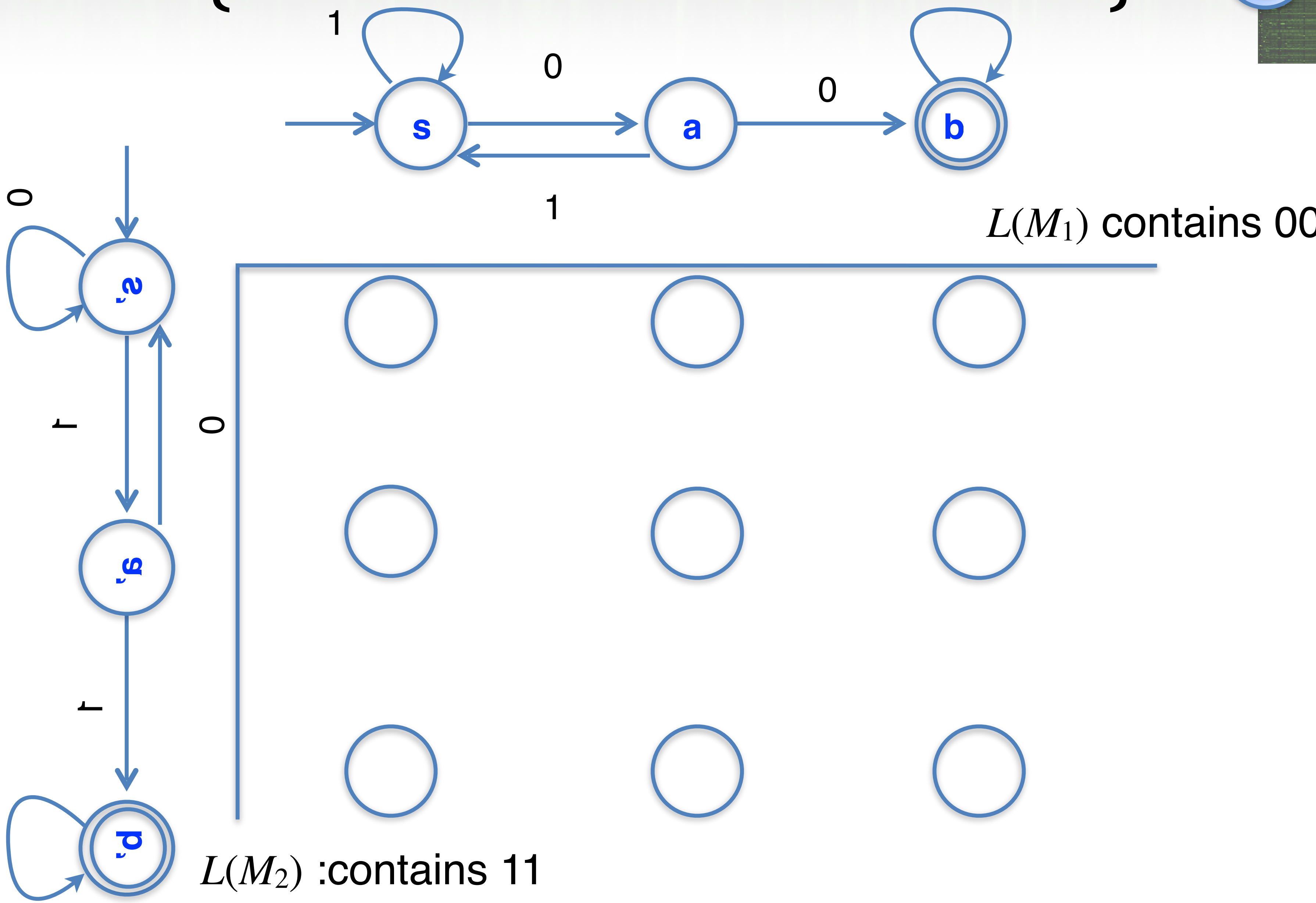
L = {w: w contains 00 and 11}?





- I want to build a machine that decides if a string contains two zeroes in a row AND two ones in a row.
- I want to run both machines at the same time.
- At the end of the string, if I am on the accept state for machine 1 AND on the accept state for machine 2, then I accept.
- How many states total?

L = {w: w contains 00 and 11}?



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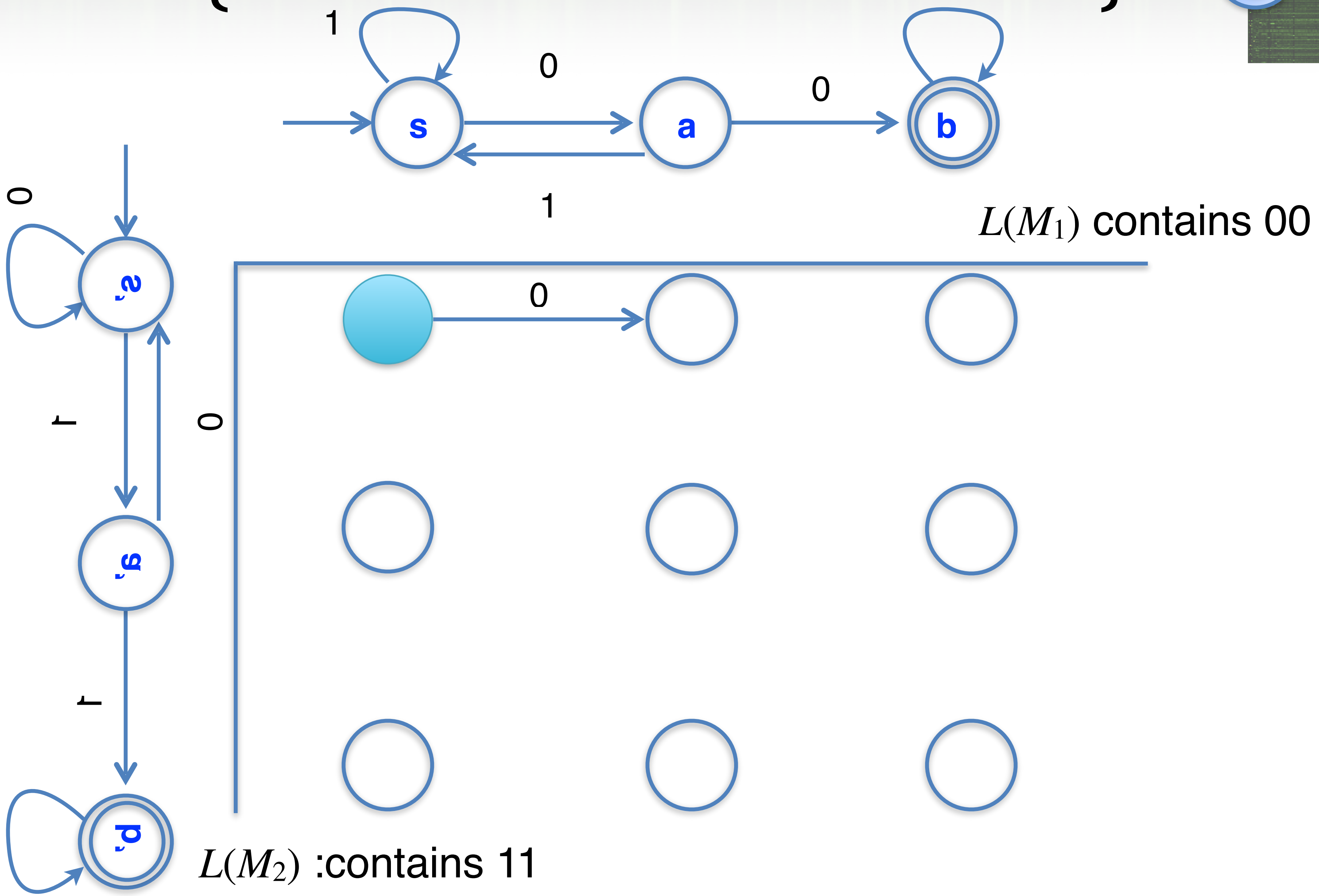

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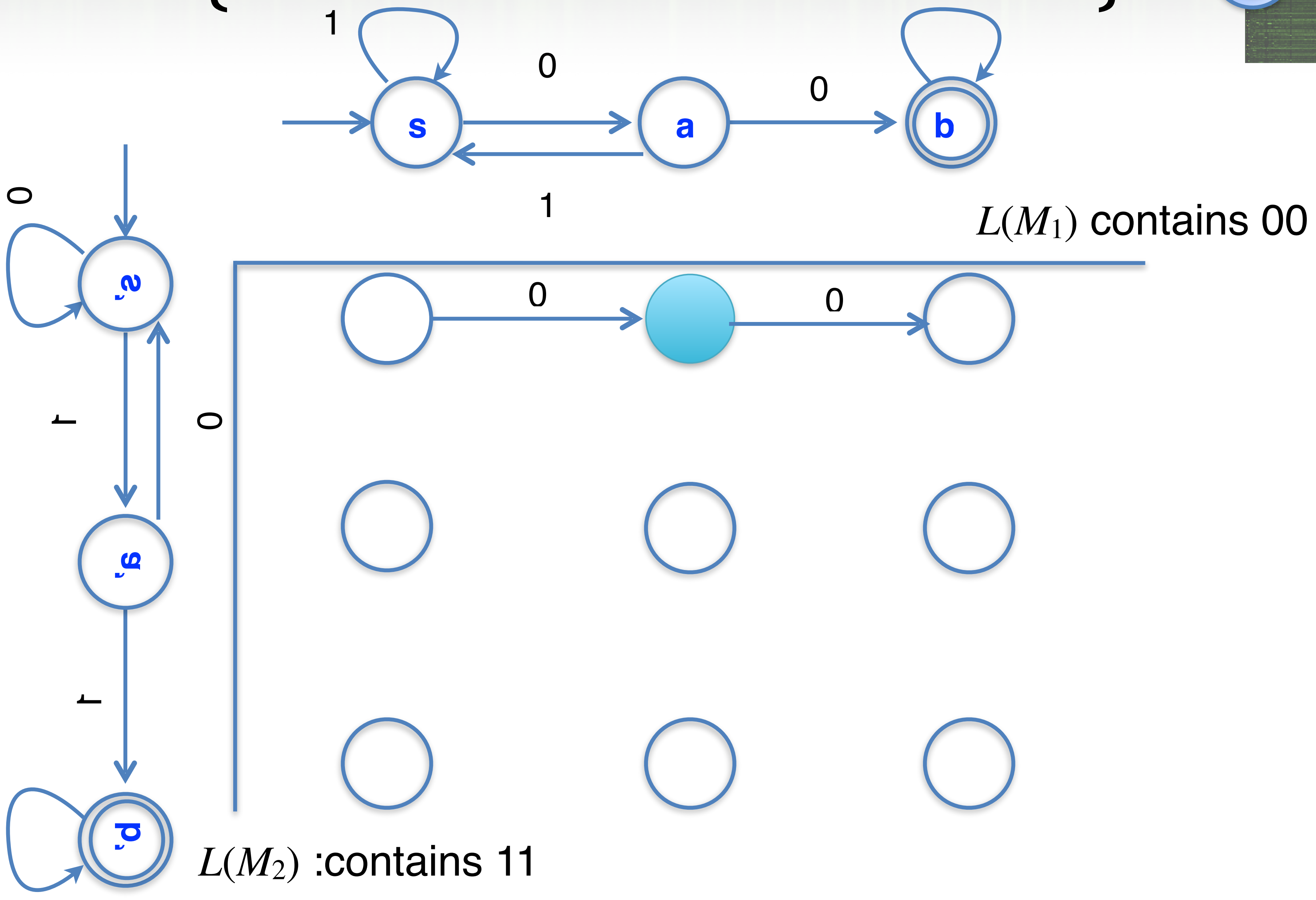

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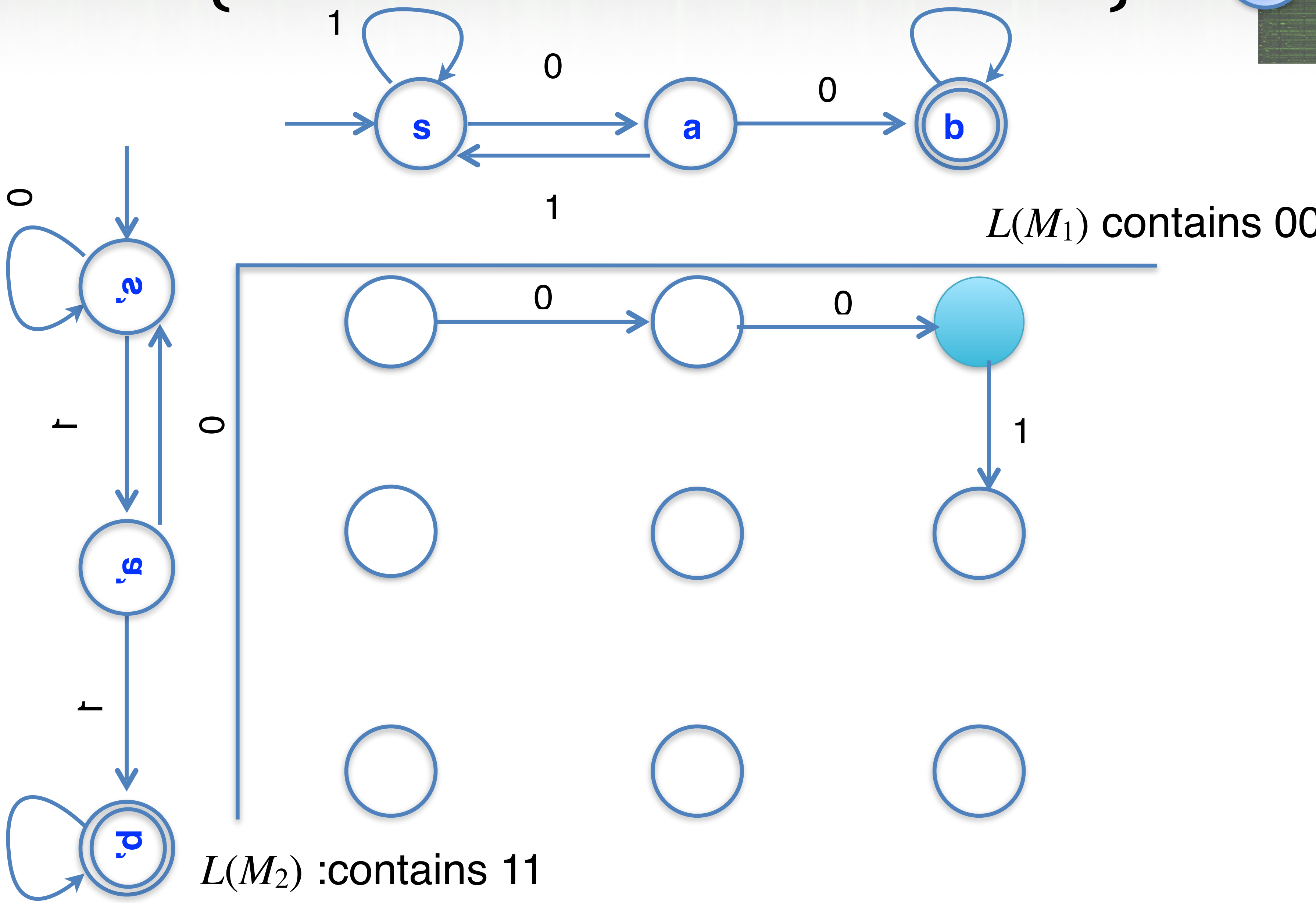

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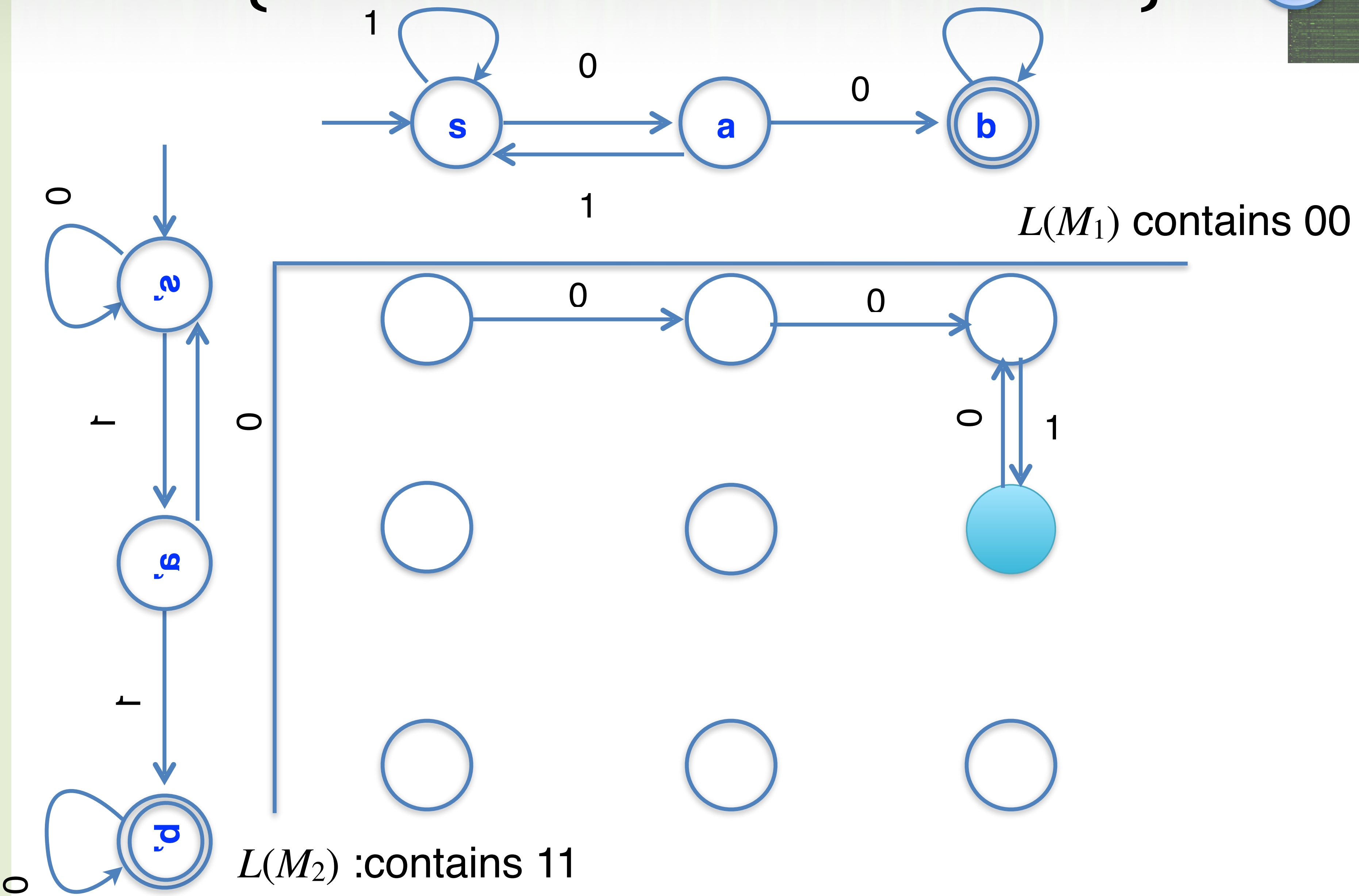

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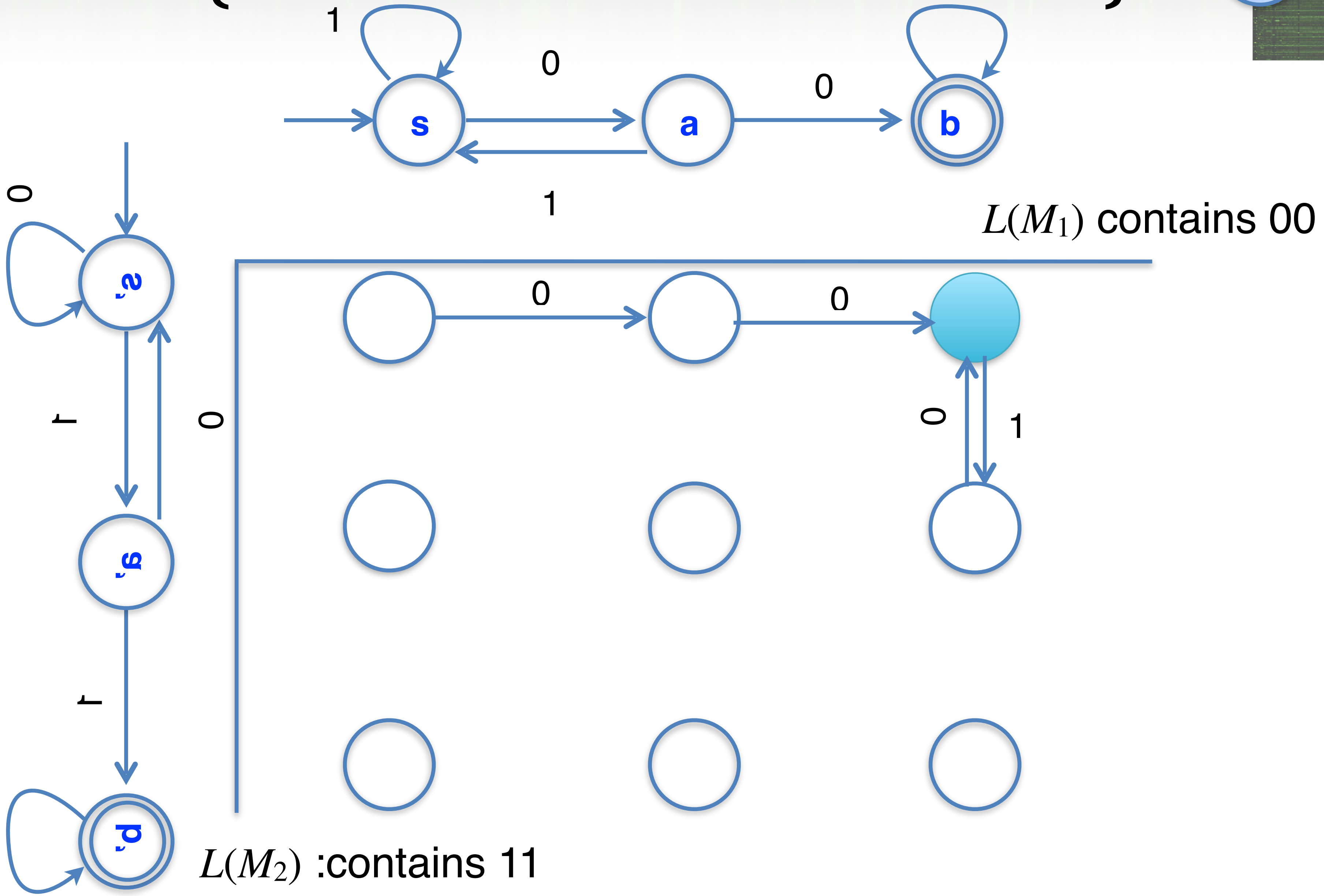

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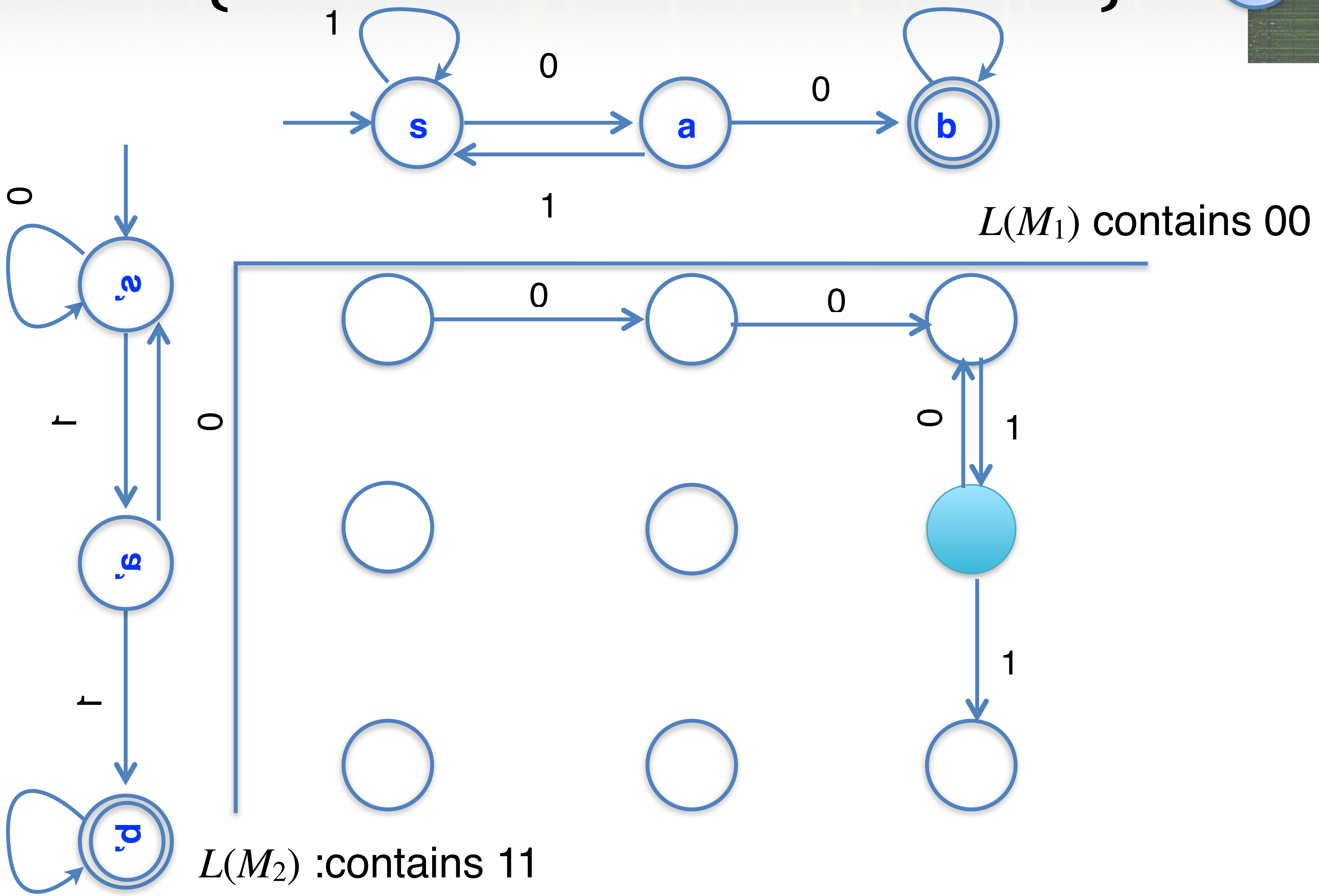

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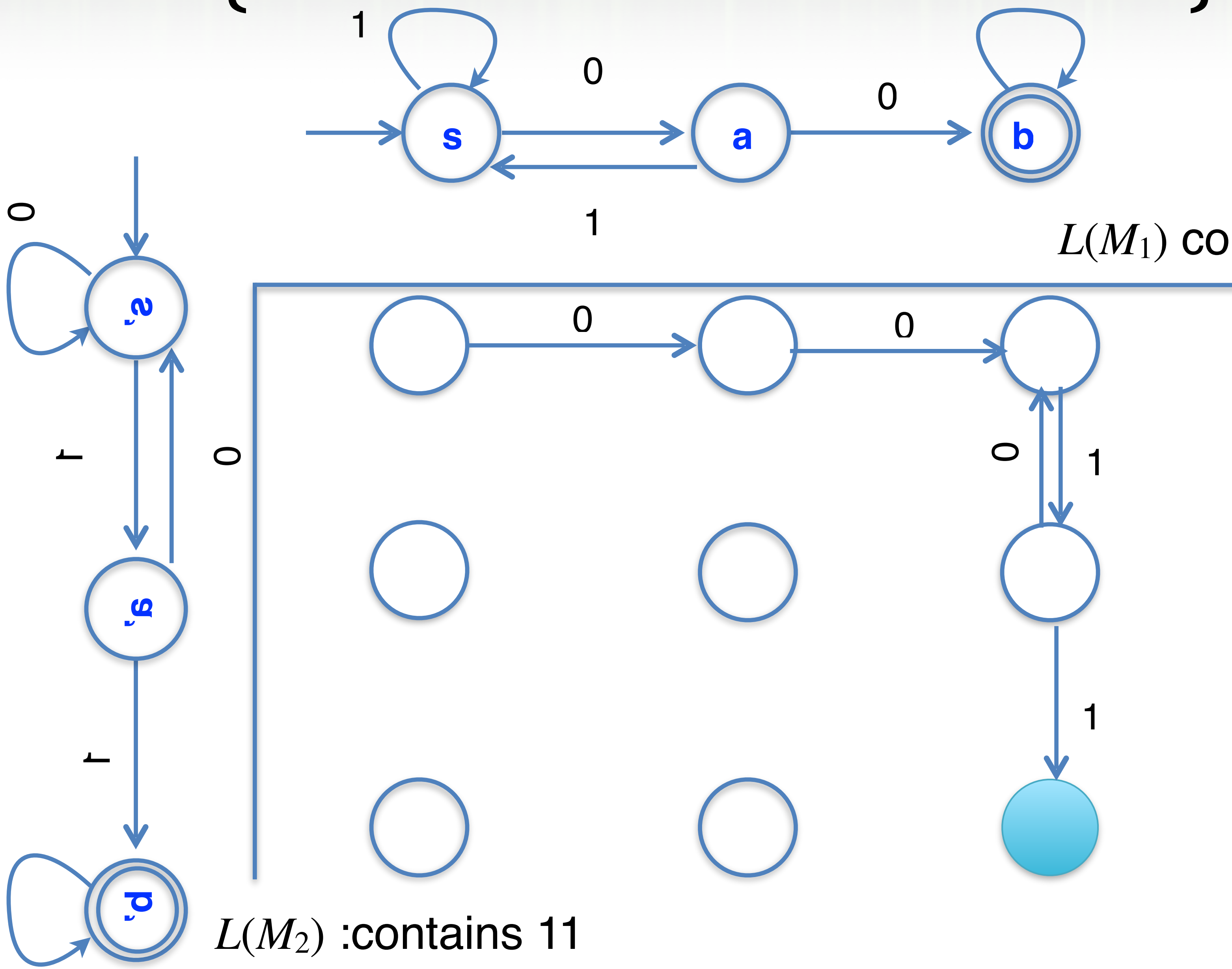

$L(M_1)$ contains 00

$L(M_2)$:contains 11

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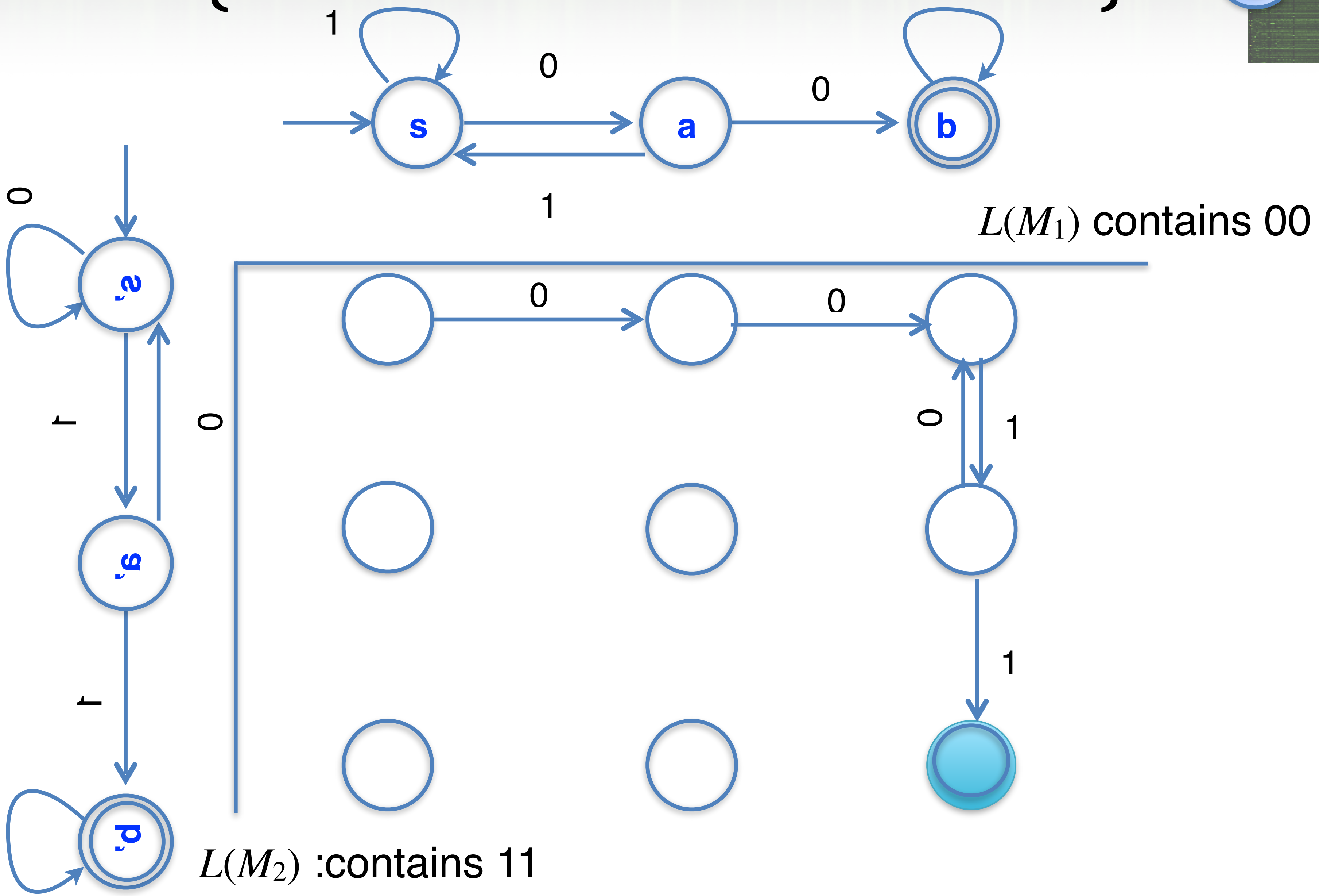

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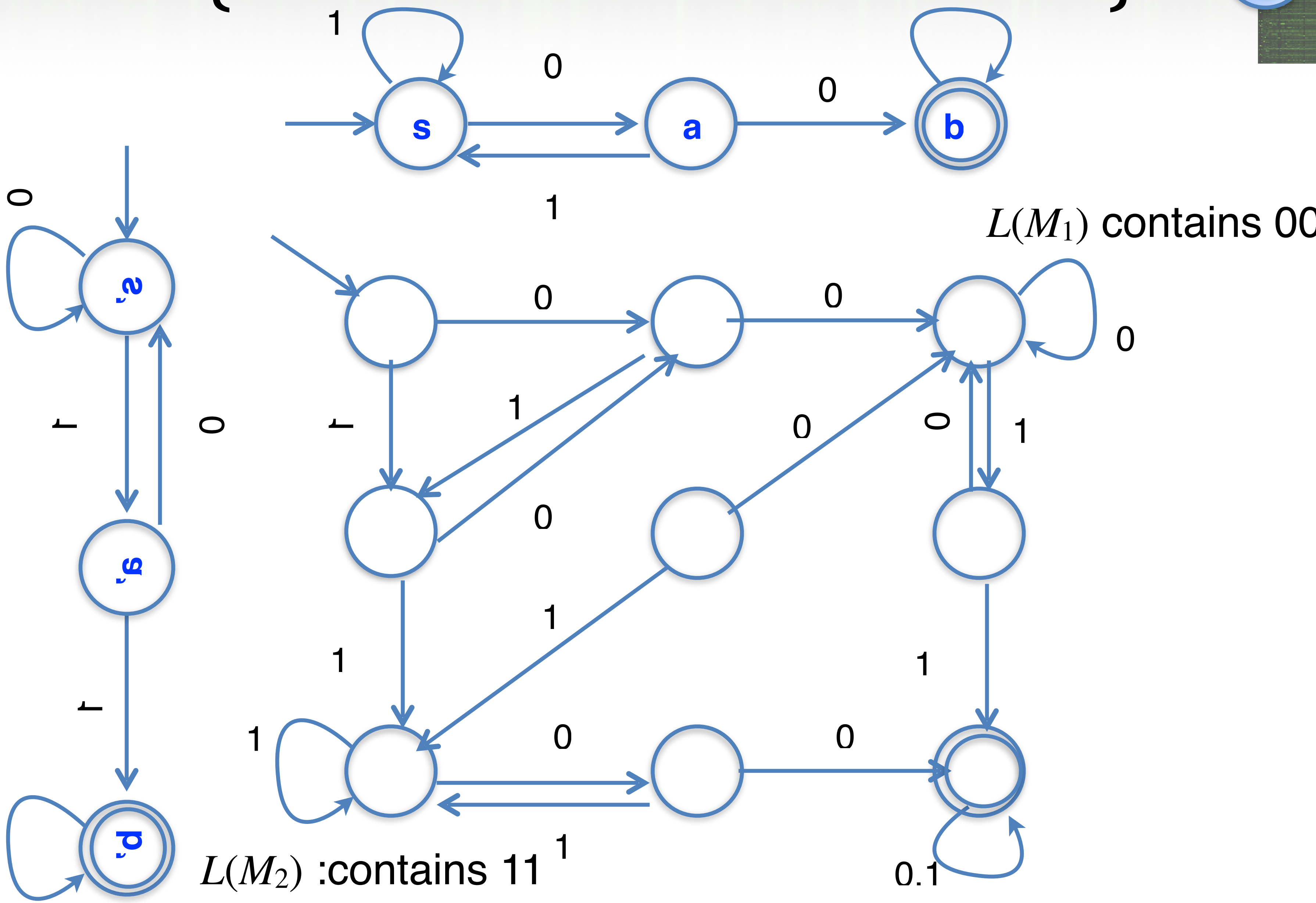

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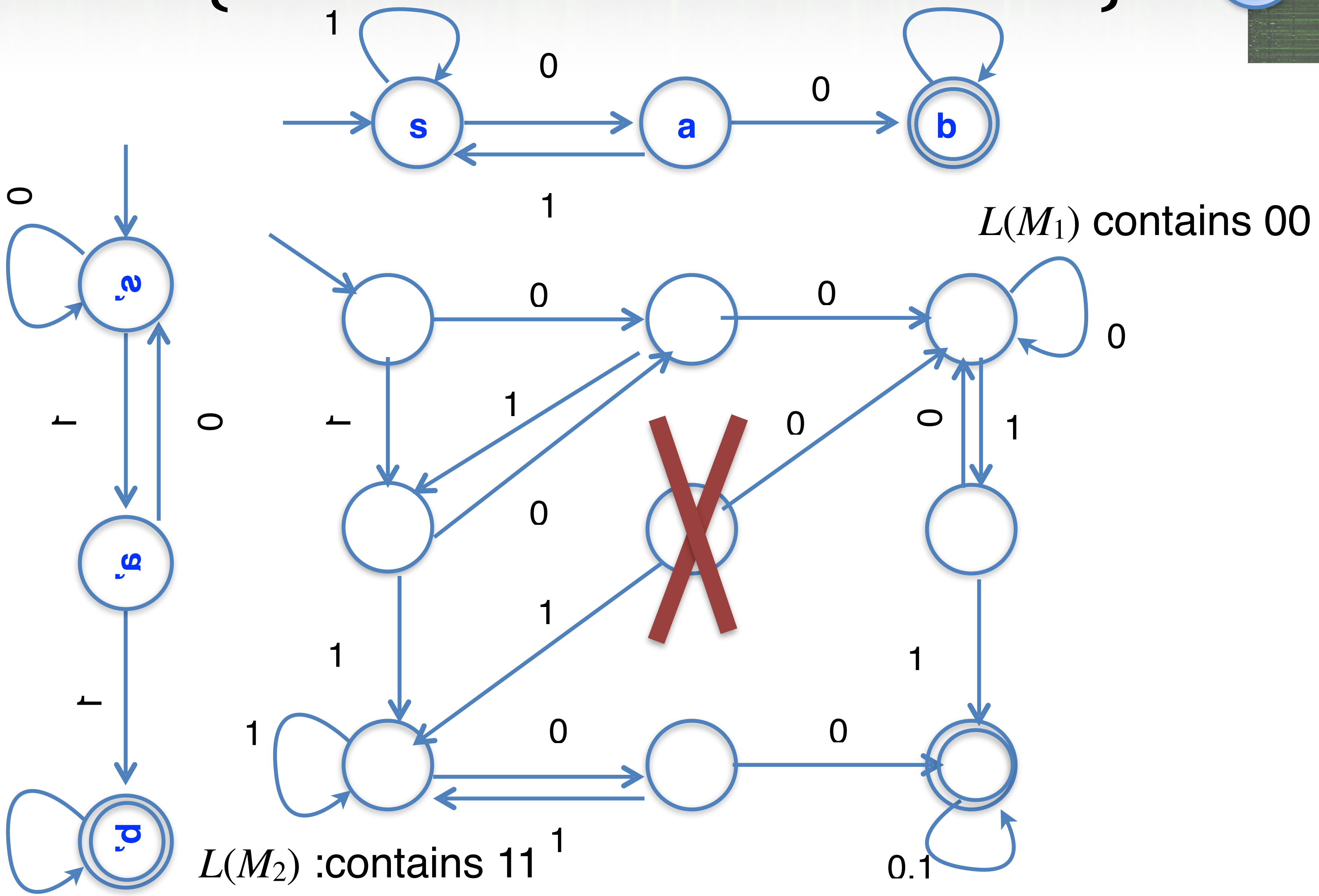

The Product Construction

Formally, given two DFAs

$M_1 = (\Sigma, Q_1, s_1, A_1, \delta_1)$ and $M_2 = (\Sigma, Q_2, s_2, A_2, \delta_2)$

Where M_1 accepts L_1

M_2 accepts L_2

$M = (\Sigma, Q, s, A, \delta)$ accepts $\mathbf{L_1 \cap L_2}$

$Q = Q_1 \times Q_2, \; s = (s_1, s_2)$

$A = \{(q_1, q_2) : q_1 \in A_1 \text{ and } q_2 \in A_2\}$

$\delta : (Q_1 \times Q_2) \times \Sigma \to Q_1 \times Q_2$

$\delta((q_1, q_2), a) = (\quad , \quad)$

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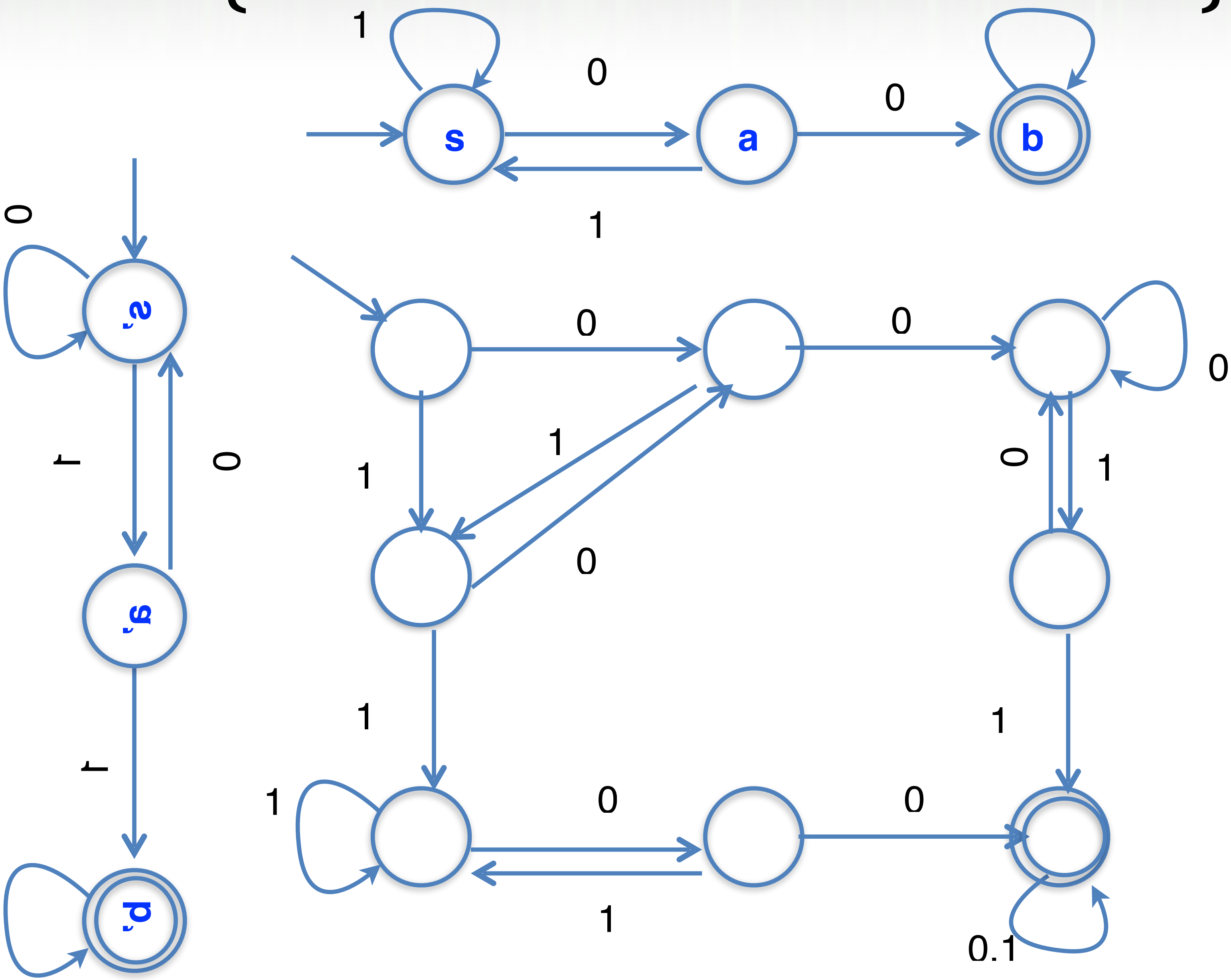

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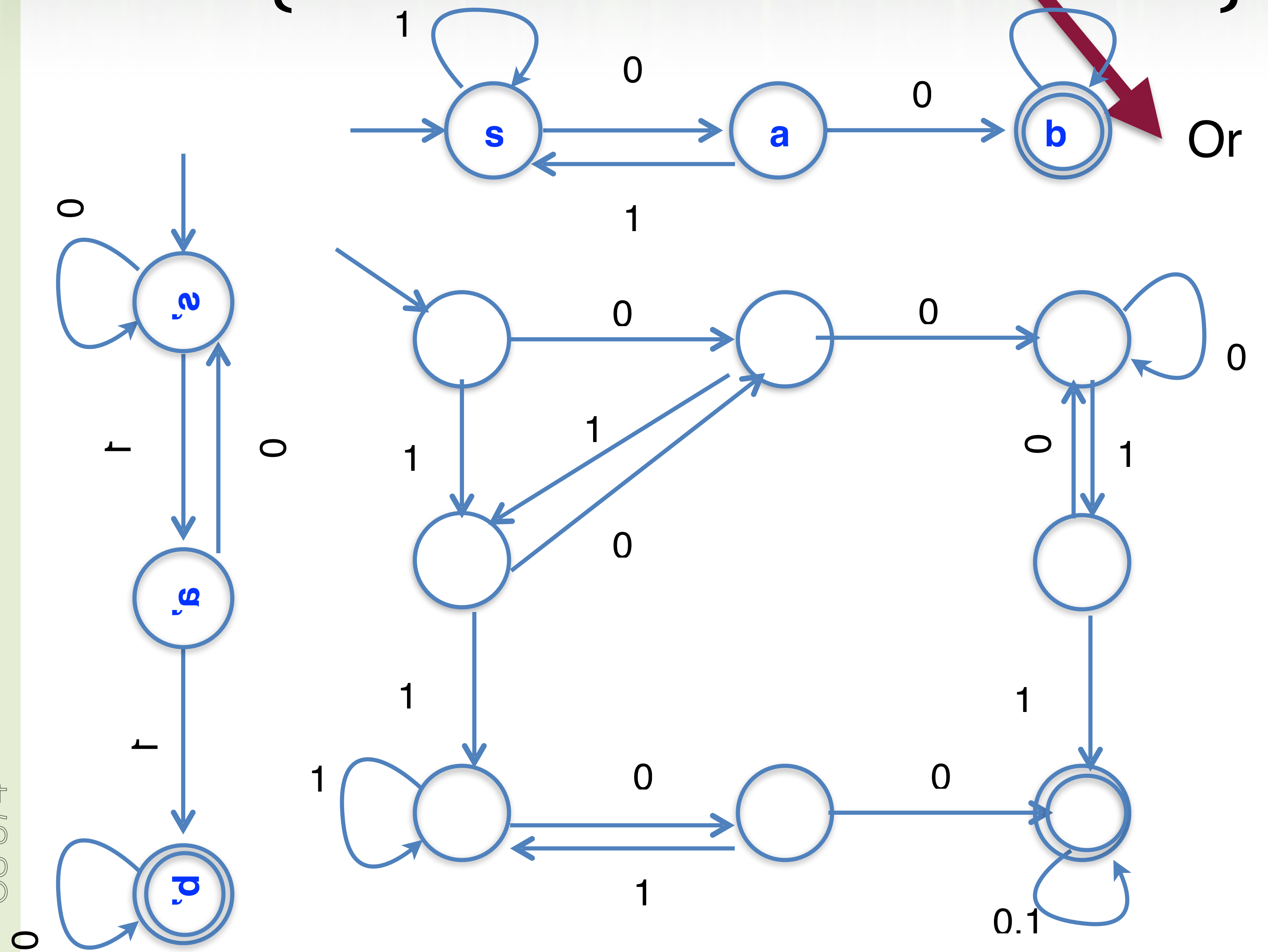

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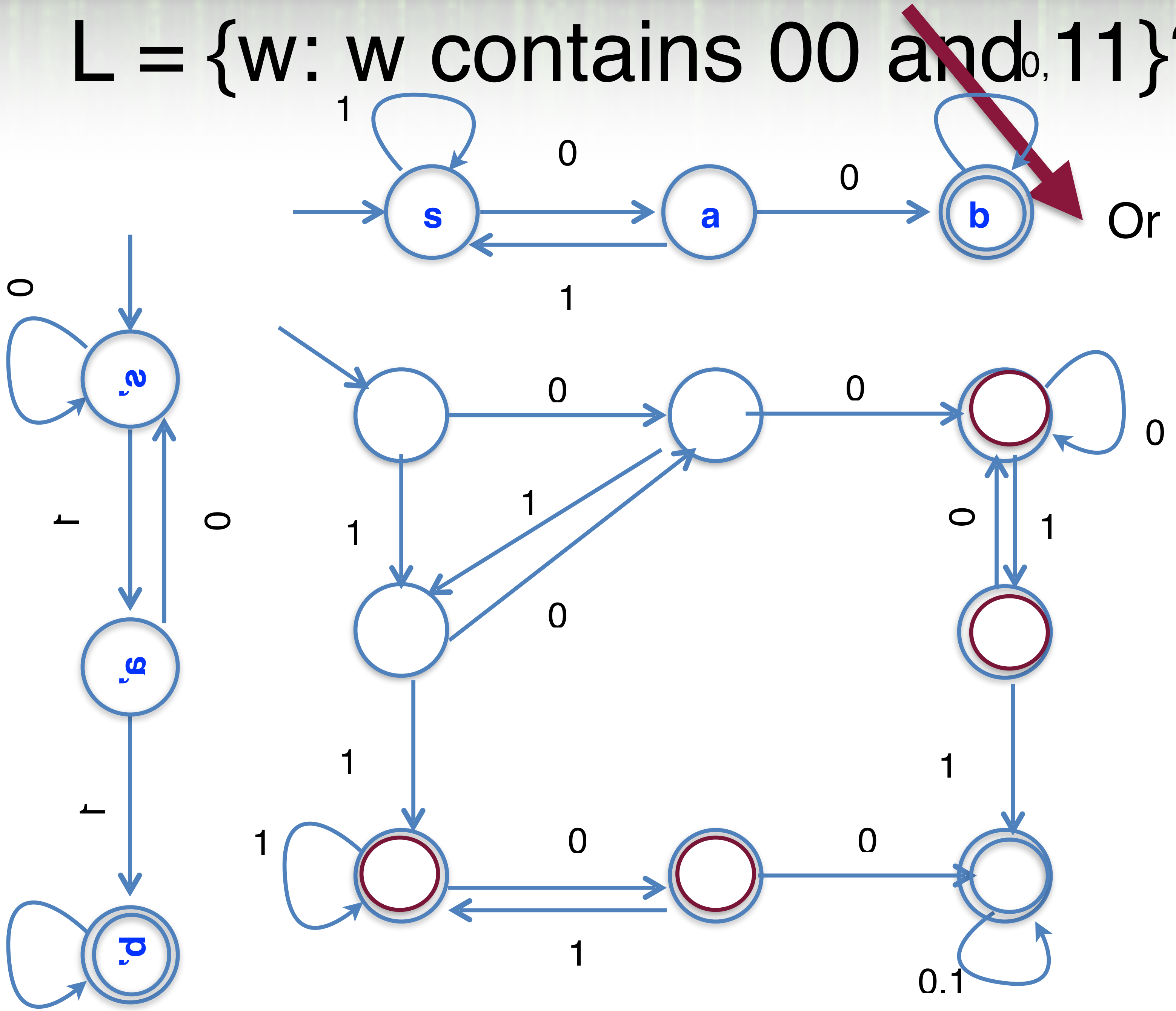

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$M = (\Sigma, Q, s, A, \delta)$ accepts $\mathbf{L_1 \setminus L_2}$

$$Q = Q_1 \times Q_2,\ \ s = (s_1, s_2)$$

$$A = \{\}\ ?$$

$$\delta: (Q_1 \times Q_2) \times \Sigma \longrightarrow Q_1 \times Q_2$$

$$\delta(\,(q_1, q_2), a) = (\quad , \quad)\ ?$$

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Closure Properties of Regular Languages

- Union: trivial for regular expressions, easy for DFAs via product
- Complement: easy for DFAs, hard for regular expressions
- Intersection: easy for DFAs via product, hard for regular expressions
- Difference: easy for DFAs via product, hard for regular expressions
- Concatenation: easy for regular expressions, hard for DFA's