Design Turing machines $M = (Q, \Sigma, \Gamma, \delta, \text{start}, \text{accept}, \text{reject})$ for each of the following tasks, either by listing the states $Q$, the tape alphabet $\Gamma$, and the transition function $\delta$ (in a table), or by drawing the corresponding labeled graph.

Each of these machines uses the input alphabet $\Sigma = \{1, \#\}$; the tape alphabet $\Gamma$ can be any superset of $\{1, \#, \&, \triangleright\}$ where $\&$ is the blank symbol and $\triangleright$ is a special symbol marking the left end of the tape. Each machine should reject any input not in the form specified below.

1. On input $1^n$, for any non-negative integer $n$, write $1^n \# 1^n$ on the tape and accept.

2. On input $\#^n 1^m$, for any non-negative integers $m$ and $n$, write $1^m$ on the tape and accept. In other words, delete all the $\#$s and shift the $1$s to the start of the tape.

3. On input $\#^n 1^n$, for any non-negative integer $n$, write $\# 2^n$ on the tape and accept. [Hint: Modify the Turing machine from problem 1.]

4. On input $1^n$, for any non-negative integer $n$, write $1^{2^n}$ on the tape and accept. [Hint: Use the three previous Turing machines as subroutines.]