We’ve seen various classes of languages that are *closed under complement*: if a language is in the class, then so is its complement. For example, the complement of a regular language is also regular. The complement of a decidable language is also decidable. The complement of a language in $\mathbf{P}$ is also in $\mathbf{P}$ (why?). However, we do not know whether or not $\mathbf{NP}$ is closed under complement. Define $\text{coNP}$ to be the set of complements of languages in $\mathbf{NP}$. In other words, $\text{coNP}$ is defined as the class of languages $L$ such that $\overline{L}$ is in $\mathbf{NP}$. The question of whether $\mathbf{NP}$ is closed under complement, is exactly the question of whether $\mathbf{NP} = \text{coNP}$, which is a question related to the question of whether or not $\mathbf{P} = \mathbf{NP}$. In these problems, we investigate some relationships between $\mathbf{NP}$ and $\text{coNP}$.

1. We saw that $\mathbf{NP}$ is the class of languages of the form $L = \{ x \mid \exists w \in \{0, 1\}^{p(|x|)} \text{ s.t. } (x, w) \in L_0 \}$ for some language $L_0$ in $\mathbf{P}$ and some polynomial $p$. Express any language $L$ in $\text{coNP}$ in terms of a language $L_1 \in \mathbf{P}$, similar to the above expression for languages in $\mathbf{NP}$.

2. Suppose $\mathbf{NP} \subseteq \text{coNP}$. Show that then $\mathbf{NP} = \text{coNP}$.

3. Show that if any $\mathbf{NP}$-complete problem is in $\text{coNP}$, then $\mathbf{NP} = \text{coNP}$.

   Hint: You may use the fact that if $L \in \text{coNP}$ and there is a polynomial time reduction from $L'$ to $L$, then $L' \in \text{coNP}$. Think about this part at home.