1. Design a one-tape TM that computes the function \( f(n) = 2n \) when \( n \) is given in unary notation. More specifically, design a TM with input alphabet \( \{0, 1, $\} \), that on input \( 0^n \) changes the tape contents to \( 0^{2n} \) and halts. On halting, the “Instantaneous Description” (or “ID”) of your TM should be \( q_{halt}0^{2n} \). (The initial ID is \( q_{start}0^n \).)

Could this be done more easily with a two-tape TM?

Can you modify your solution to obtain a TM such that \( q_{start}0^n \Rightarrow^* q_{halt}0^{2n} \) (i.e., without the $ symbol in the input or output).

2. Give a reasonably detailed description of a TM that computes the exponential function to the base 2 for unary input: i.e., it should convert an input string of the form \( 0^n \) to the string \( 0^{2^n} \). That is, you should ensure that \( q_{start}0^n \Rightarrow^* q_{halt}0^{2^n} \). You don't have to completely design the TM; just provide enough detail that a TM programmer would know what states and transitions to use. Multiple tapes will be convenient.

3. Think about at home... How would a (multi-tape) TM compute \( \lceil \log n \rceil \)? That is, if the initial ID is \( q_00^n \), then the final ID should be \( q_{halt}0^{\lceil \log n \rceil} \)