This lab covers the subset construction to convert an NFA to a DFA that accepts the same language and also on how to use the power of NFAs to prove closure under some non-trivial operations.

1. Consider the NFA defined by the transition table below, with p the initial state, and r the only accepting state.

state	input 0	input 1
p	$\{p,s\}$	$\{q\}$
q	$\{r,s\}$	$\{q\}$
r	$\{r\}$	$\{s\}$
s	$\{\ \}$	$\{q\}$

Construct a DFA that accepts the same language, using the techniques discussed in class. The states of the deterministic machine should be sets of states of the nondeterministic machine. Put your answers in the table below. *Be sure to indicate somewhere what the accepting states are, and which states are unreachable.* If it helps you might try drawing the NFA and then the DFA, but with possibly 16 states, your picture could get very confusing.

state	input 0	input 1
$\{\}$		
$\{p\}$		
$\{q\}$		
$\{r\}$		
$\{s\}$		
$\{p,q\}$		
$\{p,r\}$		
$\{p,s\}$		
$\{q,r\}$		
$\{q,s\}$		
$\{r,s\}$		
$\{p,q,r\}$		
$\{p,q,s\}$		
$\{p,r,s\}$		
$\{q,r,s\}$		
$\{p,q,r,s\}$		

2. Fix some finite alphabet Σ. For a given language L define the following three languages:

$$PREFIX(L) = \{u \in \Sigma^* \mid \exists w \in \Sigma^* \text{ such that } uw \in L\}$$

$$SUFFIX(L) = \{u \in \Sigma^* \mid \exists w \in \Sigma^* \text{ such that } wu \in L\}.$$

$$MID(L) = \{y \mid \exists x, z \in \Sigma^* \text{ such that } xyz \in L\}.$$

Show that $PREFIX(L)$, $SUFFIX(L)$ and $MID(L)$ are regular if L is regular. A useful technique here is to construct an NFA N that accepts each of these languages assuming that there is a DFA M that accepts L. More concretely, assume $M = (Q, \Sigma, \delta, q_0, F)$. Describe an NFA $N = (Q', \Sigma, \delta', q_0', F')$ where each of Q', δ', q_0', F' are defined in terms of Q, δ, q_0, F and potentially some additional information.