This lab is about strings and regular expressions. Recall the definition and properties of the concatenation operator between strings.

**Lemma 1:** Concatenating nothing does nothing: For every string $w$, we have $w \cdot \varepsilon = w$.

**Lemma 2:** Concatenation adds length: $|w \cdot x| = |w| + |x|$ for all strings $w$ and $x$.

**Lemma 3:** Concatenation is associative: $(w \cdot x) \cdot y = w \cdot (x \cdot y)$ for all strings $w$, $x$, and $y$.

1. Strings over the alphabet $\{0, 1\}$ are called boolean strings. For a boolean string $w$, define the bitwise complement $c(w)$ inductively as follows: $c(\varepsilon) = \varepsilon$, $c(0) = 1$, $c(1) = 0$, and $c(au) = c(a)c(u)$. Reversal is defined as always: $r(au) = r(u)a$ with base case $r(\varepsilon) = \varepsilon$.

Prove that $r(c(w)) = c(r(w))$ for all strings $w$. You can assume a lemma that says for all $u, v$, $c(uv) = c(u)c(v)$.

2. Give regular expressions that describe each of the following languages over the alphabet $\{0, 1\}$. We won’t get to all of these in section.

   (a) All strings containing at least three $0$s.

   (b) All strings containing at least two $0$s and at least one $1$.

   (c) All strings containing the substring $000$.

   (d) All strings not containing the substring $000$.

   (e) All strings in which every run of $0$s has length at least 3.

   (f) Every string except $000$. [*Hint: Don’t try to be clever.*]

   (g) All strings $w$ such that in every prefix of $w$, the number of $0$s and $1$s differ by at most 1.

   *(h)* All strings $w$ such that in every prefix of $w$, the number of $0$s and $1$s differ by at most 2.

   *(i)* All strings in which the substring $000$ appears an even number of times.

   *(For example, $0001000$ and $0000$ are in this language, but $000000$ is not.)*