“CS 374” Fall 2015 — Homework 0

Due Tuesday, September 1, 2015 at 10am

• • • Some important course policies • • •

- **Each student must submit individual solutions for this homework.** You may use any source at your disposal—paper, electronic, or human—but you ***must*** cite ***every*** source that you use. See the academic integrity policies on the course web site for more details. For all future homeworks, groups of up to three students will be allowed to submit joint solutions.
- **Submit your solutions in Moodle.** See instructions on the course website and submit a separate pdf for each problem.
- **Submit your solutions on standard printer/copier paper.** If you plan to typeset your homework, you can find a LaTeX template on the course web site; well-typeset homework will get a small amount of extra credit. If you hand write your home work make sure that the pdf scan is easy to read. Illegible scans will receive no points.
- **Avoid the Three Deadly Sins!** There are a few dangerous writing (and thinking) habits that will trigger an automatic zero on any homework or exam problem. Yes, we are completely serious.
  - Give complete solutions, not just examples.
  - Declare all your variables.
  - Never use weak induction.
- Unlike previous editions of this and other theory courses we are not using the “I don’t know” policy.

See the course web site for more information.

If you have any questions about these policies,
please don’t hesitate to ask in class, in office hours, or on Piazza.

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1. Prove that for any positive integer n, and any set $X \subseteq \{1, 2, \cdots, 2n\}$ such that $|X| = n + 1$, there exist two distinct elements a, b in X such that a is a multiple of b.

   [Hint: Use the pigeonhole principle. You may use the fact that any integer can be written as the product of an odd number and a power of 2.]

2. Let $u, v \in \mathbb{R}^2$ be two vectors in the real plane. Recursively define a set $L_n \subseteq \mathbb{R}^2$ as follows.

   - $L_0 = \{u, v, 0\}$. (0 denotes the zero vector in $\mathbb{R}^2$.)
   - $L_n = \{x - y \mid x, y \in L_{n-1}\}$, for $n \in \mathbb{Z}^+$.

   Let $L = \bigcup_{n=0}^{\infty} L_n$. Also, let $D = \{au + bv \mid a, b \in \mathbb{Z}\}$.

   (a) Prove that $D \subseteq L$, by giving, for each $a, b \in \mathbb{Z}$, an explicit value of n such that $au + bv \in L_n$. (You don't need to minimize the value of n; but you must argue why $au + bv \in L_n$ for your choice of n.)

   (b) Use mathematical induction to prove that for all integers $n \geq 0$, $L_n \subseteq D$, and hence $L \subseteq D$.