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## PROBLEM SET 5

### CS 373: THEORY OF COMPUTATION

Assigned: February 21, 2013    Due on: February 28, 2013

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**Instructions:** This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submissions not following these guidelines will not be graded.

**Recommended Reading:** Lecture 9, 10 and 11.

**Problem 1.** [Category: Proof] Let  $C = \{1^k x \mid x \in \{0, 1\}^*, k \geq 1, \text{ and } x \text{ contains at most } k \text{ 1s}\}$ . Using the pumping lemma, prove that  $C$  is not regular. [10 points]

**Problem 2.** [Category: Comprehension+Design] Let  $L = \mathbf{L}(1^*0(00 \cup 01 \cup 1)(0 \cup 1)^*)$ .

1. List all the equivalence classes of  $\equiv_L$ . Prove that your answer is correct. [5 points]
2. Draw the minimum state DFA  $M^L$  accepting  $L$ . [5 points]

**Problem 3.** [Category: Comprehension+Proof] For a language  $L \subseteq \Sigma^*$ , define an equivalence  $\simeq_L$  on  $\Sigma^*$  as follows

$$x \simeq_L y \text{ iff } \forall z. zx \in L \leftrightarrow zy \in L$$

Notice that this is a slightly different equivalence than  $\equiv_L$  defined in Lecture 11. Prove that  $L$  is regular iff  $\simeq_L$  has finitely many equivalence classes. *Hint:* Can you see a connection between  $\simeq_L$  and  $\equiv_{L^R}$ , where  $L^R$  refers to the reverse of  $L$ ? [10 points]