Problem Set 2
CS 373: Theory of Computation

Assigned: January 24, 2013   Due on: January 31, 2013

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lectures 3 and 4.

Problem 1. [Category: Design+Proof] Let $A_k \subseteq \{a, b\}^*$ be the collection of strings w where there is a position i in w such that the symbol at position i (in w) is a, and the symbol at position $i + k$ is b. For example, consider A_2 (when $k = 2$). $baab \in A_2$ because the second position ($i = 2$) has an a and the fourth position has a b. On the other hand, $bb \notin A_2$ (because there are no as) and $aba \notin A_2$ (because none of the as are followed by a b 2 positions away).

1. Design a DFA for language A_k. Your formal description (by listing states, transitions, etc. and not "drawing the DFA") will depend on the parameter k but should work no matter what k is; see lecture 2, last page for such an example. **[5 points]**
2. Prove that your DFA is correct when $k = 2$. **[5 points]**

Problem 2. [Category: Comprehension] Consider the following NFA M_0 over the alphabet $\{0, 1\}$.

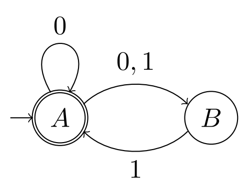

Figure 1: NFA M_0 for Problem 2

1. Describe formally what the following are for automaton M_0: set of states, initial state, final states, and transition function. **[4 points]**
2. What are $\hat{\delta}_{M_0}(A, 010)$, $\hat{\delta}_{M_0}(A, 101)$, $\hat{\delta}_{M_0}(A, 1101)$, and $\hat{\delta}_{M_0}(B, 10)$? **[4 points]**
3. What is $\mathbf{L}(M_0)$? You don't have to prove your answer. **[2 points]**

Problem 3. [Category: Design+Proof] Consider the language $A_2 \subseteq \{a, b\}^*$, from problem 1, which was defined to be the collection of strings w where there is a position i in w such that the symbol at position i (in w) is a, and the symbol at position $i + 2$ is b.

1. Design an NFA for language A_2 that has at most 4 states. You need not prove that your construction is correct, but the intuition behind your solution should be clear and understandable. **[5 points]**

2. Prove that any DFA recognizing A_2 has at least 5 states. **[5 points]**