Problem Set 1
CS 373: Theory of Computation

Assigned: January 17, 2013 Due on: January 24, 2013

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded. The website `www.automatatutor.com` might be useful to solve problem 3. It is a tool to teach students how to design DFAs for languages. It has an easy user interface where you can draw automata for problems and the tool will either say that your solution is correct, or provide examples to help you fix your incorrect solution. The languages in problem 3 are posed as "problems" on the website. However, the website is only available until Monday, January 21, midnight.

Recommended Reading: Lectures 1 and 2 and website `http://www.automatatutor.com`

Problem 1. [Category: Comprehension+Proof]

1. Let $A = \{1, 2, 3\}$, $B = \{\emptyset, \{1\}, \{2\}\}$, and $C = \{1, 2, \{1, 2\}\}$. Compute $A \cup B$, $A \cap B$, $B \cap C$, $A \cap C$, $A \times B$, $A \times C$, $C \setminus A$, $C \setminus B$, $A \times B \times C$, and 2^B. Recall that 2^A denotes the *power set* of A, and $A \setminus B$ denotes A *set difference* B. **[5 points]**
2. Prove for any two sets A and B, $A \times B = B \times A$ if and only if $A = B$ or $A = \emptyset$ or $B = \emptyset$. **[5 points]**

Problem 2. [Category: Comprehension] Consider the following DFA M_0 over the alphabet $\{0, 1\}$.

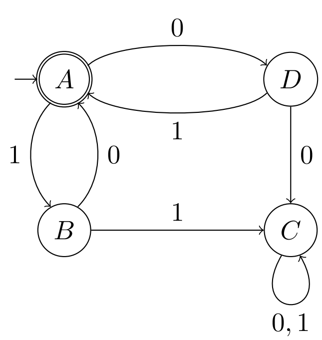

Figure 1: DFA M_0 for Problem 2

1. Describe formally what the following are for automaton M_0: set of states, initial state, final states, and transition function. **[4 points]**
2. What are $\hat{\delta}_{M_0}(A, \epsilon)$, $\hat{\delta}_{M_0}(A, 1011)$, $\hat{\delta}_{M_0}(B, 101)$, and $\hat{\delta}_{M_0}(C, 10110)$? **[4 points]**
3. What is $\mathbf{L}(M_0)$? **[1 points]**

4. What is the language recognized if we change the initial state to B? What is the language recognized if we change the set of final states to be $\{B\}$ (with initial state A)? **[1 points]**

Problem 3. [Category: Design] Design DFAs to recognize the following languages; the alphabet in each case is $\{a, b\}$. You need not prove that your construction is correct, but your construction must be clear and understandable.

1. $L_1 = \{w \in \{a,b\}^* \mid w \text{ starts with } a \text{ and has an odd number of } ab \text{ substrings}\}$. For example, $aaab \in L_1$ (has only 1 ab), $bab \notin L_1$ (does not begin with a), and $abbab \notin L_1$ (has 2 ab substrings). **[5 points]**
2. $L_2 = \{w \in \{a,b\}^* \mid w \text{ starts with } ab \text{ or } |w| \text{ is not divisible by } 3\}$. For example, $abb \in L_2$ (because it begins with ab), $baba \in L_2$ (because $|baba| = 4$ is not divisible by 3), and $bab \notin L_2$. **[5 points]**