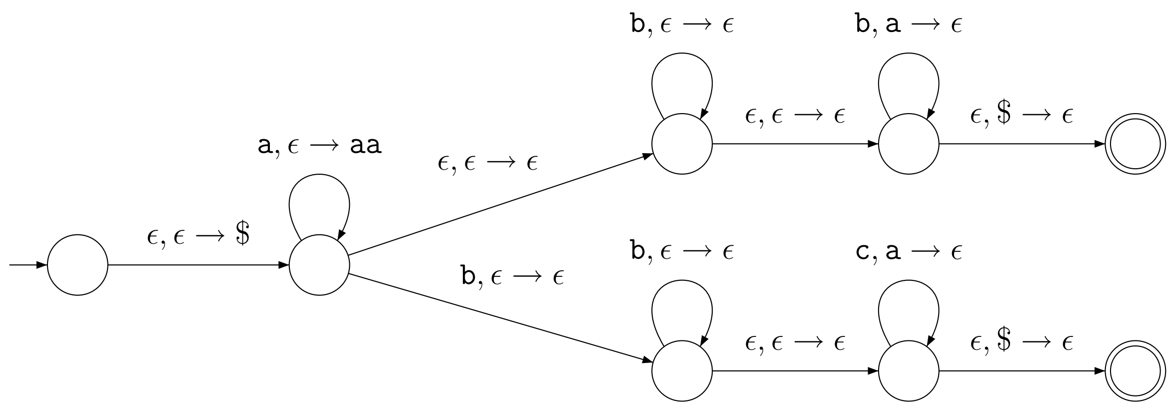
CS 273: Intro to Theory of Computation, Spring 2008
Problem Set 8
Due Tuesday, March 11th, 10am

This homework contains five problems, one of which is bonus. Please submit each on a **separate sheet of paper**. This will help us grade your homeworks more quickly. Turn in your homework at Elaine Wilson's office (3229 Siebel).

1. Extract language from PDA.

   Give the language of the following PDA.



2. Converting CFG to PDA.

   For each of the following languages (with alphabet $\Sigma = \{\texttt{a}, \texttt{b}\}$) construct a pushdown automaton recognizing that language, following the general construction for converting a context-free grammar to a PDA (lecture 13, pp. 115–118 in Sipser).

   For each language, also give a parse tree for the word w, a leftmost derivation for w, and the first 10 configurations (state and stack contents) for the PDA as it accepts w.

   a) The word $w = \texttt{aababbaabbbb}$ and the grammar with start symbol S:

   $S \to \texttt{a}TT\texttt{b}$
   $T \to \epsilon \mid T\texttt{ab} \mid \texttt{a}T\texttt{b} \mid \texttt{b}$

   b) The word $w = \texttt{babaabaabaaba}$ and the grammar with start symbol S:

   $S \to AB \mid B$
   $A \to BB \mid B$
   $B \to CC \mid \texttt{b}$
   $C \to \texttt{aba}$

3. Language to PDA.

   Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and consider the language

$$L = \left\{ \mathtt{a}^i \mathtt{b}^j \mathtt{c}^k \;\middle|\; i \neq j \text{ or } j \neq k \right\}.$$

   Design a PDA for L. Present your PDA as a state diagram, with brief comments about how it works.

4. Context-free grammar design.

   Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

   A pair of strings (x, w) is an S-pair if they are identical except that two characters have been swapped. Formally, if $x = c_1 c_2 \ldots c_n$ and $w = d_1 d_2 \ldots d_n$, then there are two character positions i and j such that $c_i = d_j$ and $c_j = d_i$, and $c_k = d_k$ for k other than i or j.

   (a) Notice that for any word $x \in \Sigma^*$, it holds that (x, x) is always a S-pair. Briefly explain why.

   (b) Let $L_S = \left\{ wx^R \;\middle|\; x, w \in \Sigma^* \text{ and } (x, w) \text{ is an S-pair} \right\}$.

   Give a context-free grammar that generates L_S.

   (c) Let

$$L_T = \left\{ w_1 \# w_2 \# \ldots w_i \# \;\middle|\; \begin{array}{l} w_i \in \Sigma^* \text{ for all } i \text{ and} \\ \exists i, j \text{ such that } i < j \text{ and } (w_i, w_j^R) \text{ is an } S\text{-pair} \end{array} \right\}.$$

   Give a context-free grammar that generates L_T.

5. Context-free grammar. (Bonus)

   The following grammar (with start symbol S) is ambiguous:

$$S \Rightarrow \mathtt{a}S \mid \mathtt{a}S\mathtt{b}S \mid \epsilon$$

   (a) Show that the grammar is ambiguous, by giving two parse trees for some string w.

   (b) Give an efficient test to determine whether a string w in $L(S)$ is ambiguous. Explain informally why your test works.