CS 273: Intro to Theory of Computation, Spring 2008
Problem Set 6 (due Monday, February 25th, 4pm)

This homework contains four problems, one of which is bonus. Please submit each on a **separate sheet of paper**. This will help us grade your homeworks more quickly. Turn in your homework at Elaine Wilson's office (3229 Siebel).

1. Pumping lemma problems:

   (a) Use pumping lemma to prove that L is not regular, where:

   $$L = \{a^k b^m : k \leq m \text{ or } m \leq 2k\}$$

   (b) Prove that the following language satisfies pumping lemma:

   $$L = \{00, 11\}$$

   From the fact that it satisfies the pumping lemma, can we deduce that L is regular? Why or why not?

2. Decide of the following languages are regular. If they are regular, give a DFA,NFA or regular expression for the language. If it is not regular, then give a proof using either closure properties or the pumping lemma.

   (a) $L_1 = \{xy \mid x \text{ has the same number of } a\text{'s as } y\}$

   (b) $L_2 = \{w \mid w \text{ has three times the number of } a\text{'s than } b\text{'s}\}$

   (c) $L_3 = \{xy \mid x \text{ has the same number of } a\text{'s as } y \text{ and } |x| = |y|\}$

3. Let T be the language $\{0^n 1^n : n \geq 0\}$. Use closure properties to show that the following languages are not regular, using a proof by contradiction and the fact that T is known not to be regular.

   (a) $L = \{a^n b^m c^{n+m} : n \geq m \geq 0\}$

   (b) $J = \{0^n 1^n 2^n : n \geq 1\}$

4. (bonus) Show that if L is regular then $t(L)$ is regular where:

   $$t(L) = \{x : \text{for some string } y, |y| = |x| \text{ and } xy \in L\}$$

   so t is an operation that preserves regularity.