CS 273: Intro to Theory of Computation, Spring 2008
Problem Set 4 (due Monday, February 11th, 4pm)

This homework contains four problems. As usual, please submit each problem on a **separate sheet of paper**. Turn in your homework at Elaine Wilson's office (3229 Siebel).

1. NFA DESIGN/SUBSET CONSTRUCTION.

   (a) Design an NFA for the following language:

   $$L = \left\{ x \;\middle|\; x \text{ is a binary string that has 1101 or 1100 or 0001 as a substring} \right\}.$$

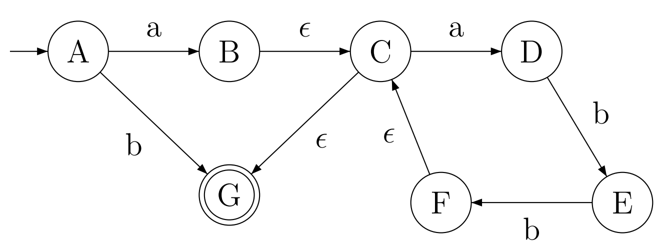
   Design this initial NFA in a modular way, using ϵ-transitions.

   (b) Remove all ϵ-transitions from your NFA. (Namely, present an equivalent NFA with no ϵ-transitions.)

   (c) Convert your ϵ-free NFA into a DFA using subset construction.

2. NFA INTERPRETATION/FORMAL DEFINITIONS.

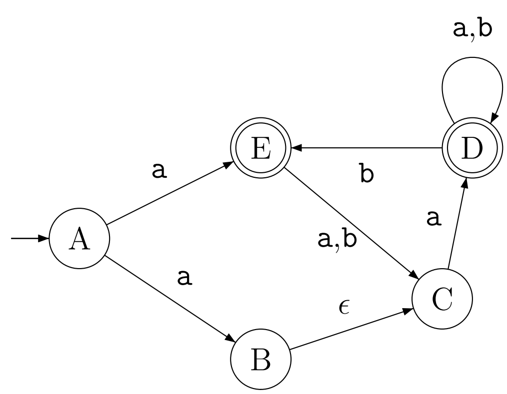
   Consider the following NFA M.



   (a) Give a regular expression that represents the language of M. Explain briefly why it is correct.

   (b) Recall the definition of an NFA accepting a string w (Sipser p. 54). Show formally that M accepts the string $w = \texttt{aabb}$

   (c) Let $\Sigma = \{\texttt{a}, \texttt{b}\}$. Give the formal definition of the following NFA N.



3. NFA design with guessing.

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and define the language L

$$L = \left\{ x_1 \# x_2 \# \ldots \# x_n \;\middle|\; \forall i, x_i \in \Sigma^2, \text{ and } \exists i, j \text{ such that } i \neq j \text{ and } x_j = x_k \right\}.$$

That is, each x_i is a string of two characters from Σ. And two of the x_i's need to be identical, but you don't know which two are identical. So the language contains $\mathtt{ab\#bb\#cc\#ab}$ and $\mathtt{ac\#bb\#ac\#ab}$, but not $\mathtt{aa\#ac\#bb}$.

Design an NFA that recognizes L. This NFA should "guess" when it is at the start of each matching string and verify that its guess is correct.

4. NFA modification.

The 2SWP operation on strings interchanges the character in each odd position with the character in the following even position. That is, if the string length k is even, the string $w_1 w_2 w_3 w_4 \ldots w_{k-1} w_k$ becomes $w_2 w_1 w_4 w_3 \ldots w_k w_{k-1}$. E.g. abcbac becomes babcca. If the string has odd length, we just leave the last (unpaired) character alone. E.g. abcba becomes babca.

Given a whole language L, we define $\mathsf{2SWP}(L)$ to be $\left\{ \mathsf{2SWP}(w) \;\middle|\; w \in L \right\}$.

Show that regular languages are closed under the 2SWP operation. That is, show that if L is a regular language, then $\mathsf{2SWP}(L)$ is regular. That is, suppose that L is recognized by some DFA M. Explain how to build an NFA N which accepts $\mathsf{2SWP}(L)$.