CS 273: Intro to Theory of Computation, Spring 2008
Problem Set 13
Due Tuesday, April 22nd, 10am

This homework contains five problems, one of which is bonus. Please submit each on a **separate sheet of paper**. Turn in your homework at Elaine Wilson's office (3229 Siebel).

1. Language classification.

   Suppose that we have a set of Turing machine encodings defined by each of the following properties. That is, we have a set

   $$L = \left\{ \langle M \rangle \;\middle|\; M \text{ is a } \mathsf{TM} \text{ and } M \text{ has property } P \right\},$$

   and we are considering different ways to fill in P. Assume that the Turing machines M have only a single tape.

   (a) P is "there is an input string which M accepts after no more than 327 transitions."

   (b) P is "on blank input, M halts leaving the entire tape blank."

   (c) P is "M's code has no transitions into the reject state."

   (d) P is "on input UIUC, M never changes the contents of the even-numbered positions on its tape." (That is, it can read the even-numbered positions, but not write a different symbol onto them.)

   For each of these languages, determine whether it is Turing decidable, Turing recognizable, or not Turing recognizable. Briefly justify your answers.

2. Reduction I.

   Define the language L to be

   $$L = \left\{ \langle M \rangle \;\middle|\; M \text{ is a } \mathsf{TM},\ L(M) \text{ is context free but is not regular} \right\}.$$‘

   Show that L is undecidable by reducing A_{TM} to L. (Do the reduction directly. Do not use Rice's Theorem.)

3. Reduction II.

   Define the language L to be

   $$L = \left\{ M \;\middle|\; M \text{ is a } \mathsf{TM} \text{ and } 100 \leq |L(M)| \leq 200 \right\}.$$

   Show that L is undecidable by reducing A_{TM} to L. (Do the reduction directly. Do not use Rice's Theorem.)

4. Interleaving.

   Suppose that we have Turing machines M and M' which enumerate languages L and L', respectively.

(a) Describe how to construct an enumerator P for the language $L \cup L'$. The code for P will need to make use of the code for M and M' (e.g. call it as a subroutine or run it in simulation using U_{TM}).

(b) Suppose that the languages L and L' are infinite and suppose that M and M' enumerate their respective languages in lexicographic order. Explain how to modify your construction from part (a) so that the output of P is in lexicographic order.

5. Confusing but Interesting(?) Reduction. (bonus)

Reduce L to A_{TM} (notice the different direction of reduction, in particular don't reduce A_{TM} to L):

$$L = \left\{ \langle M, w \rangle \;\middle|\; \text{execution of } M \text{ with input } w \text{ never stops} \right\}.$$