CS 273: Intro to Theory of Computation, Spring 2008
Problem Set 12
Due Tuesday, April 15th, 10am

This homework contains three problems. Please submit each on a **separate sheet of paper**. Turn in your homework at Elaine Wilson's office (3229 Siebel).

1. Decidable problems.

   Prove that L is a decidable language:

   $$L = \left\{ \langle D, k \rangle \;\middle|\; \begin{array}{l} D \text{ accepts no string of length } \leq k, \\ \text{and } D \text{ is a } \mathsf{NFA} \end{array} \right\}.$$

2. Enumerators I.

   An enumerator for a language L is a Turing machine that writes out a list of all strings in L. See p. 152–153 in Sipser.

   The enumerator has no input tape. Instead, it has an output tape on which it prints the strings, with some sort of separator (e.g. #) between then. The strings can be printed in any order and duplicates of the same string are ok. But each string in L must be printed eventually.

   Design an enumerator that writes all tuples of the form (n, p) where $n \in \mathbb{N}$, $p \in \mathbb{N}$, and n is a multiple of p.

3. Enumerators II.

   If L and J are two languages, define $L \oplus J$ to be the language containing all strings that are in exactly one of L and J. That is

   $$L \oplus J = \{w \mid w \in L \text{ and } w \notin J \text{ or } w \in J \text{ and } w \notin L\}$$

   Suppose that you are given two context-free grammars G and H.

   (a) Design an enumerator that will print all strings in $L(G) \oplus \text{Ł}(H)$.

   (b) Is $L(G) \oplus \text{Ł}(H)$ context-free? TM recognizable? TM decidable? Briefly justify your answer.

   (c) Recall that

   $$EQ_{CFG} = \left\{ \langle G, H \rangle \;\middle|\; G \text{ and } H \text{ are } \mathsf{CFG}\text{'s and } L(G) = L(H) \right\}.$$

   We have mentioned in class that EQ_{CFG} is undecidable. Why is this problem harder than the ones you just solved in parts (a) and (b)?