CS 273: Intro to Theory of Computation, Spring 2008
Problem Set 11
Due Tuesday, April 8th, 10am

This homework contains four problems. Please submit each on a **separate sheet of paper**. Turn in your homework at Elaine Wilson's office (3229 Siebel).

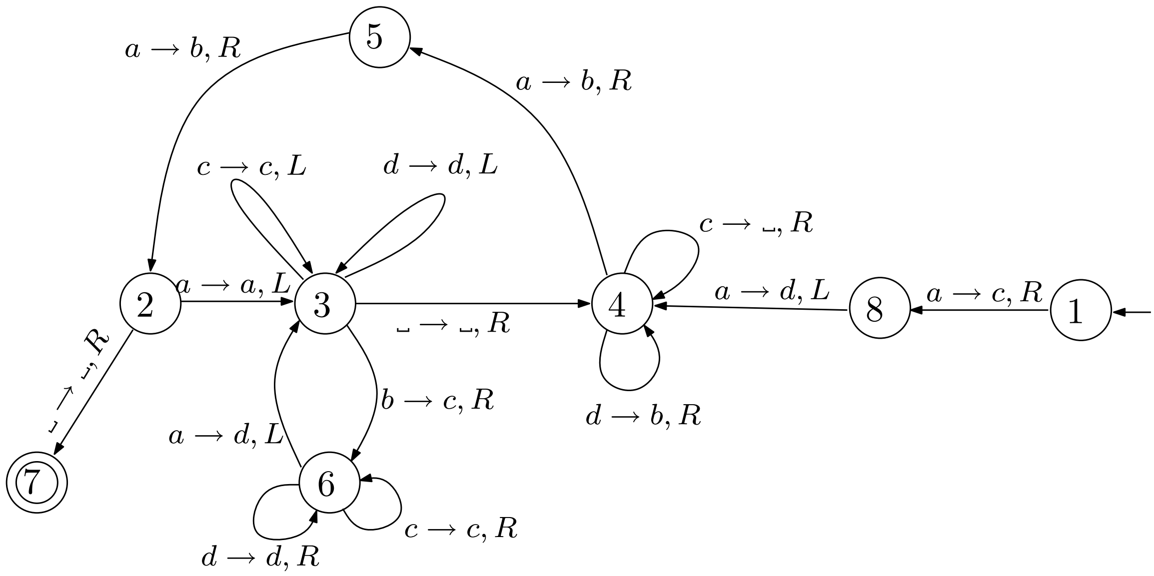
1. Turing machine tracing.

   In the following Turing machine $\Sigma = \{\mathtt{a}\}$ and $\Gamma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{B}\}$.

   (a) What is the language of this TM?

   (b) Informally and briefly explain why this TM accepts the language you claimed in the previous part.

   (c) Trace the execution of this TM as it processes string aaaaaaaaa (i.e., a sequence of 9 as) by providing the sequence of configurations it goes through (i.e., tape & state in each step - use the configuration notation shown in class).



2. High Level TM Design.

   Design a Turing Machine that for a given input tape with n cells containing 0's, marks the positions which are composite numbers. Specifically, cells at the prime-numbered positions are left containing 0, cells at composite-numbered locations are left containing X, and the cell at the first (leftmost) location is left containing U (for unit). For example, consider the input 0000000000. This represents the first 10 numbers. So the Turing machine should halt with U00X0X0XXX on the tape.

3. Turing machine encodings.

In this problem we demonstrate a possible encoding of a TM using the alphabet $\{\texttt{0}, \texttt{1}, ;, |\}$ where $|$ is the newline character. We encode $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$ as a string $n|i|j|t|r|s|w$ where n, i, j, t, r, s are integers in binary representing $|Q|$, $|\Sigma|$, $|\Gamma|$, q_0, q_{acc}, q_{rej} and w represents Σ as described below. We adopt the convention that states are numbered from 0 to $n-1$, the input alphabet symbols are numbered from 0 to $i-1$, and the tape alphabet symbols are numbered from 0 to $j-1$ with $j-1$ representing the special blank symbol (therefore $j > i$).

The string w represents δ as follows. Each transition $(q, \texttt{a}) \to (q', \texttt{b}, D)$ is represented as a 5-tuple $q; \texttt{a}; q'; \texttt{b}; D$ where $q, \texttt{a}, q', \texttt{b}$ are integers in binary and D is either 0 (move left) or 1 (move right). We adopt the convention that only useful transitions are represented and any transition not represented leads to the reject state. The string w consists of 5-tuples separated by $|$. Thus $w = w_1|w_2|...|w_p$ where p is the number of useful transitions.

Here is the representation of a mystery Turing machine M, using this encoding. For ease of reading, we have shown $|$ as an actual line break and given the integers in decimal rather than binary.

$$
\begin{array}{c}
8 \\
2 \\
3 \\
7 \\
3 \\
5 \\
7;1;0;2;1 \\
0;1;0;1;1 \\
0;0;1;1;1 \\
1;1;1;1;1 \\
1;0;0;0;1 \\
0;2;2;2;0 \\
6;2;3;2;1 \\
2;1;2;1;0 \\
2;0;6;0;0 \\
6;1;6;1;0 \\
6;0;4;0;0 \\
4;0;4;0;0 \\
4;1;4;1;0 \\
4;2;0;2;1
\end{array}
$$

(a) Draw a state diagram for this TM (omitting the reject state).

(b) What is the language of this TM? Give a brief justification.

4. Turing machine simulation.

A PlaneTM is a TM that instead of a tape (line), uses a quadrant of the plane for recording. Consider all the points in the plane with coordinates $P = \left\{ (x, y) \;\middle|\; x, y \in \mathbb{Z}, \quad x, y \geq 0 \right\}$. We assume that each point of P indicates a memory cell in the plane and a PlaneTM has a head that can change the content of the cell that it is on top of it and after that the head transfers to a neighbor cell.

Formally a PlaneTM is $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$, where Q is the set of states, Σ and Γ are the input and tape alphabets, $q_0, q_{\text{acc}}, q_{\text{rej}}$ are the initial, accept and reject states, respectively. The initial position of the head is always cell $(0, 0)$. The input sequence is written from left to right on the cells $(0, 0), (1, 0), (2, 0), \cdots$; and the rest of cells are filled with $\textvisiblespace$ (blank symbol, $\textvisiblespace \in \Gamma$). Transition function, $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{\mathtt{U}, \mathtt{D}, \mathtt{L}, \mathtt{R}\}$, where U (resp. D) indicates that the head should be moved up (resp. down).

If $\delta(q, \mathtt{a}) = (q', \mathtt{b}, D)$ then the PlaneTM moves from state q when seeing character a under the tape head, to the state q', replace character a with character b. Furthermore, the head is moved to the cell above/below/left/right of the current cell if D is $\mathtt{U}, \mathtt{D}, \mathtt{L}, \mathtt{R}$, respectively.

As with our normal Turing machines, the head simply stays put if the commanded move would take it off the edge of the quadrant.

When a PlaneTM powers up, its input (which contains no blanks) occupies a rectangular region in the lower left corner. The other cells (i.e. the infinite areas to the right and above the input) are filled with blanks.

(a) Show that every TM can be simulated using a PlaneTM.

(b) Show that every PlaneTM can be simulated using a TM.

From this, we can conclude that TM's and PlaneTM's are equivalent.