CS 273: Intro to Theory of Computation, Spring 2008
Problem Set 1 (due Tuesday, January 22nd, 4pm)

This homework contains five problems. Please follow the homework format guidelines posted on the class web page. In particular, submit each problem on a **separate sheet of paper**, put your name on each sheet, and write your discussion section time (e.g. 10) in the upper righthand corner. These details may sound picky, but they make the huge pile of homeworks much easier to grade quickly.

Turn in your homework at Elaine Wilson's office (3229 Siebel).

1. Set theory (10 points)

   Let $A = \{1, 2, 3\}$, $B = \{\varnothing, \{1\}, \{2\}\}$ and $C = \{1, 2, \{1, 2\}\}$. Compute $A \cup B$, $A \cap B$, $B \cap C$, $A \cap C$, $A \times B$, $A \times C$, $C - A$, $C - B$, $A \times B \times C$, and $\mathbb{P}(B)$. ($\mathbb{P}(B)$ is the power set of B.)

2. Sets of Strings (10 points)

   (a) Let $A = \{\text{apple, orange, grape}\}$ and $B = \{\text{green, blue}\}$. What is the set

   $$C = \{ba \mid a \in A, b \in B\}.$$

   (b) Let X be the string uiuc. List all substrings of X.

3. Induction and Strings (10 points)

   (a) Let our alphabet Σ be $\{a, b, c\}$. For any non-negative integer n, how many strings of (exactly) length n are there (over the alphabet Σ)?

   (b) Prove your claim in (a) by induction on n.

4. Recursive definitions (10 points)

   Consider the following recursive definition of a set S.

   (a) $(3, 5) \in S$
   (b) If $(x, y) \in S$, then $(x + 2, y) \in S$
   (c) If $(x, y) \in S$, then $(-x, y) \in S$
   (d) If $(x, y) \in S$, then $(y, x) \in S$
   (e) S is the smallest set satifying the above conditions.

   Give a nonrecursive definition of the set S. Explain why it is correct.

5. Set Theory (10 points)

   Suppose that A, B, and C are sets. We claim that if $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then $B = C$.

   (a) Explain informally why this is true, using words and/or a Venn diagram.

   (b) Prove it formally, using standard identities such as DeMorgan's Laws.