Problem Set 10

Submission instructions: Submit each problem on a separate sheet of paper, put your name on each sheet, and write your discussion section time and day (e.g. Tuesday 10am) in the upper righthand corner. These details may sound picky, but they make the huge pile of homeworks much easier to grade quickly and more importantly, since we return them in the discussion sections, easier for you to get them back.

Also, write on each exercise the name/netid of your group members.

Due: Friday, April 24, 2009 at 12:30 in Elaine Wilson office (SC 3229). If the door is locked, slide your solutions under the door. Version: **1.01**

(Q1) Enumerators

[Category: Construction, Points: 10]

(a) Design an enumerator that will list all positive integral solutions to a polynomial inequality in three variables, which will be given as input on the input tape of the Turing machine.

I.e., you need to give all positive integral solutions to a given inequality $P(x, y, z) \ge c$. An example of such an inequality is $2x^2y^2 + xy + z \ge 5$.

You may use pseudo code to describe your solution.

(b) Modify the enumerator above to give all integral (positive or negative) solutions.

(Q2) Decision and Enumeration.

[Category: Construction, Points: 12]

(a) Show that L_1 is a decidable language:

 $L_1 = \{ \langle M, x, i \rangle : \text{Machine } M \text{ will accept string } x \text{ in no more than } i \text{ steps} \}$

(b) Show that L_2 is a decidable language:

 $L_2 = \{ \langle M, x, i \rangle : M(x) = "yes" and head of M will use only the first i cells of its tape \}$

(c) Explain how to build an enumerator for L_3 :

 $L_3 = \{ \langle M, x \rangle : M \text{ accepts } x \text{ in no more than } 2^{|x|} \text{ steps} \}$

(Q3) Aliens in outer space.

[Category: Understanding, Points: 4]

It is widely believed by certain people that intelligent aliens exist in outer space. Some of these people will be happy to discuss seriously Invisible Pink Unicorns¹, a tendency that dramatically undercuts their credibility. On the other hand, there are people that conjecture the existence of intelligent life on earth, despite overwhelming evidence to the contrary.

One could of course try to resolve the existence of aliens in outer space question by building spacecrafts, go out, look around, etc. But this is an expensive proposition that would require much time and expense. As such, resolving the question of existence of aliens in outer space seems to be quite challenging.

A cheaper solution would be to build a TM that would resolve the question. The TM would print "Yes" on the tape if aliens in outer space exist, and then it would stop. Similarly, it aliens do not exist then it would print "No way" on the tape and stop.

Argue that the existence of aliens in outer space is a decidable problem. That is, there exists a TM that always stops and prints "Yes" iff there are aliens in outer-space.

$\left(\mathrm{Q4}\right)\,$ I am a liar.

[Category: Puzzle, Points: 4]

A town has two kinds of people, visually indistinguishable, called Grubsies and Greepies. Greepies always tell the truth; Grubsies always lie (names can be deceiving, you see).

You come to the town, and chance upon a person (who could be a Grubsy or a Greepy) and you want to find out whether a particular road leads to Wimperland (assume all people in the town know the answer to the question).

Find a single YES/NO question that you can ask the person so that you can figure out whether the road leads to Wimperland.

Grubsies and Greepies are fictional; any resemblance to person or persons living or dead or undead is purely coincidental.

Hint: think of the diagonalization proof of undecidability of the membership problem for Turing machines.

¹See http://en.wikipedia.org/wiki/Invisible_Pink_Unicorn.