## Problem Set 1

Due: Thursday, February 5 at 12:30 in class (i.e., SC 1105)

Submission instructions: Submit each problem on a separate sheet of paper, put your name on each sheet, and write your discussion section time and day (e.g. Tuesday 10am) in the upper righthand corner. These details may sound picky, but they make the huge pile of homeworks much easier to grade quickly and more importantly, since we return them in the discussion sections, easier for you to get them back.

Also, write on each exercise the name/netid of your group members.

## 1. DFA building

[Category: Construction, Points: 10]

Let  $\Sigma = \{a, b\}$ . Let L be the set of words w in  $\Sigma^*$  such that w has an even number of b's and an odd number of a's and does not contain the substring ba.

- (a) [1 Points] Give a regular expression for the language of all strings over  $\Sigma$  that do not contain ba as a substring.
- (b) [1 Points] Give a regular expression for the language L above (shorter expression is better, naturally).
- (c) [8 Points] Construct a deterministic finite automaton for L that has at most 5 states. Make sure that your DFA is complete (has transitions from all actions on all states).

If you find this hard, you can also give a DFA with more states that accepts L for partial credit.

Hint: You may want to try to enumerate some elements in L, and see whether you can simplify the description of L.

 $2. \ {\sf Control}.$ 

[Category: Construction, Points: 10]

We want to build the controller for an automatic door, that opens for some time when a sensor senses that a person is approaching the door.

Let  $\Sigma = \{tick, approach, open, close\}$ 

Every odd event is a "tick" or an "approach", and every even event is the controller's response to it, "open" or "close". An "open" event directs the door to open if it is closed, and to remain open if it is open. Similarly, a "close" event directs the door to close or remain closed. An "approach" event happens when the the sensor detects that a person is approaching the door, and a "tick" event denotes the passage of one second of time. We want the door to open immediately after detecting an "approach" event, and remain open for exactly 2 ticks after the last "approach" event, at which point the door must close. (Thus, the input has even length and is made out of pair of word. Each pair is a command, followed by current status report.)

Build an automaton that accepts all valid sequences of this controller behavior. Your automaton should be over the alphabet  $\Sigma$ , and, for example, must accept

tick.close.approach.open.tick.open.tick.open.tick.close,

and accepts

approach.open.tick.open.approach.open.tick.open.tick.open.tick.close.

But it rejects the word

tick.open,

and rejects the word

tick.close.approach.open.tick.open.tick.close.

3. Recursive definitions

[Category: Construction, Points: 10]

Consider the following recursive definition of a set S.

- (a)  $(1, 58) \in S$
- (b) If  $(x, y) \in S$  then  $(x + 2, y) \in S$
- (c) If  $(x, y) \in S$  then  $(-x, -y) \in S$
- (d) If  $(x, y) \in S$  then  $(y, x) \in S$
- (e) S is the smallest set satisfying the above conditions.

Give a nonrecursive definition of the set S. Explain why it is correct.

## 4. Set Theory

[Category: Proof, Points: 10]

For any two arbitrary sets X and Y, we have that  $(X \setminus (X \cap Y)) \cap (Y \setminus (Y \cap X))$  is an empty set.

- (a) Explain informally why this is true, using words and/or a Venn diagram.
- (b) Prove it formally.
- 5. No such thing.

[Category: Proof, Points: 10]

## (Extra credit.)

Let  $L_{\text{same}}$  be the language of all strings over  $\{0, 1, 2\}$  that have the same number of 0s as the sum of the number of 1s and 2s. Provide a formal (correct) proof that no finite automata (i.e., DFA) can accept  $L_{\text{same}}$ .