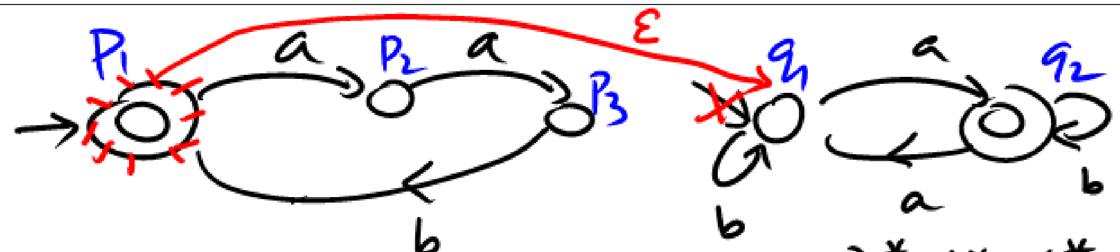
Lecture #5:

Nondeterminism

and nondeterministic finite automata.



$L_1 = (aab)^*$ $\quad$ $L_2 = (b^*ab^*a)^*\, b^*ab^*$

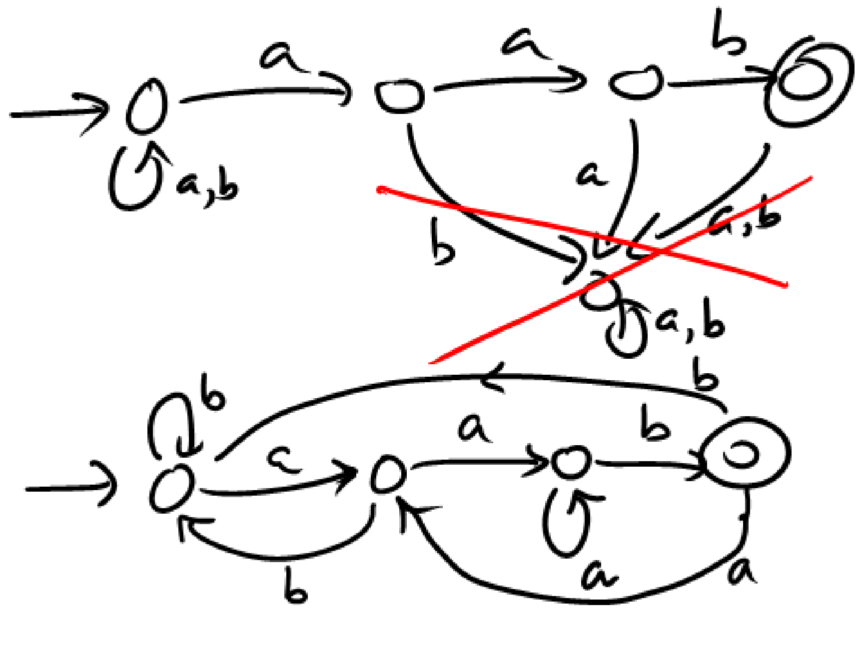
$L_1 \cdot L_2$: DFA

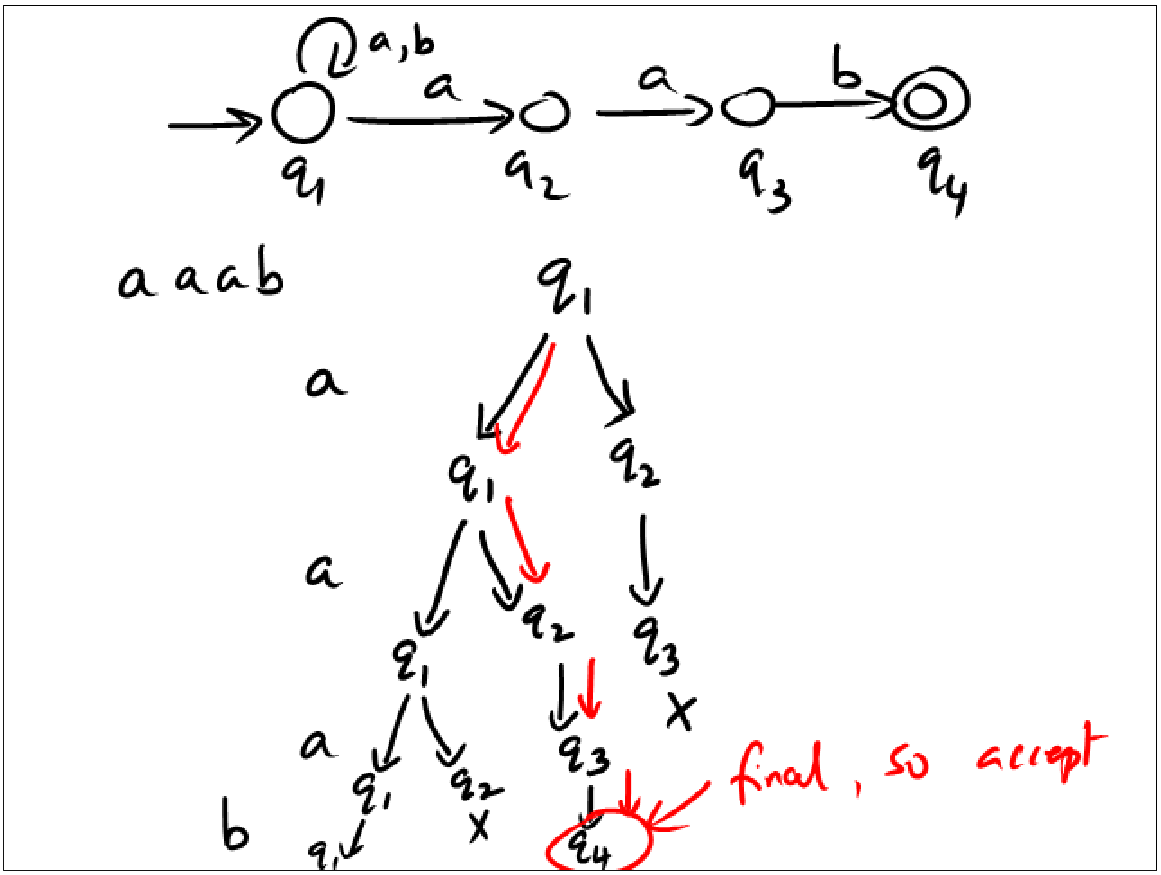
w $\quad$ w_1 | w_2

aaba

- $p_1 \xrightarrow{a} p_2 \xrightarrow{a} p_3 \xrightarrow{b} p_1 \xrightarrow{a} p_2$
- $p_1 \xrightarrow{a} p_2 \xrightarrow{a} p_3 \xrightarrow{b} p_1 \xrightarrow{\varepsilon} q_1 \xrightarrow{a} q_2$
- $p_1 \xrightarrow{\varepsilon} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_1 \xrightarrow{b} q_1 \xrightarrow{a} q_2$

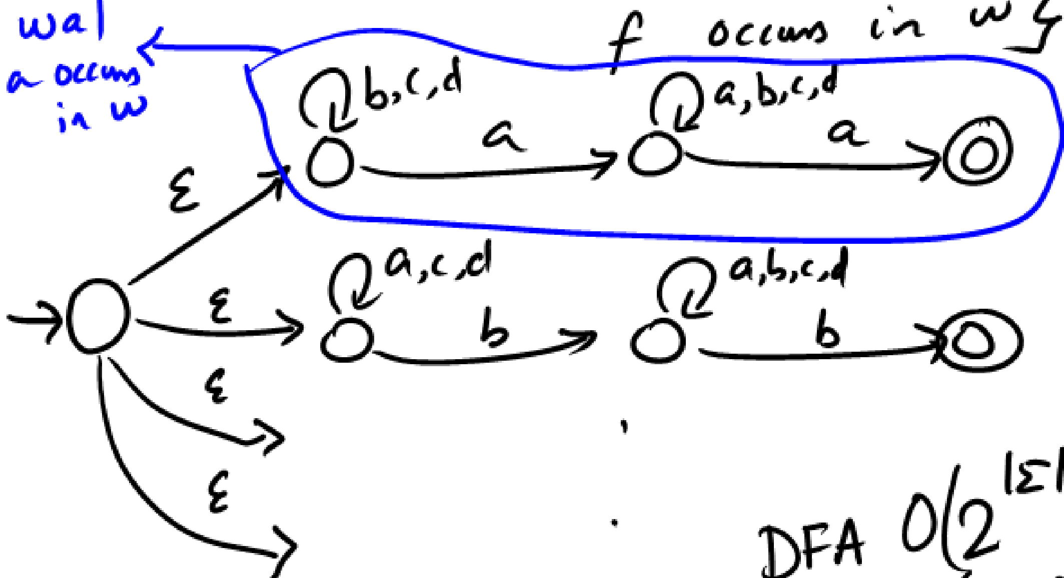
$$L = \{ waab \mid w \in \{a,b\}^* \}$$





a,b
a
a
b
q_1
q_2
q_3
q_4
aaab
a
a
a
b
X
X
final, so accept

$$L = \{ wf \mid w \in \{a,b,c,d\}^*,\ f \in \{a,b,c,d\},\ f \text{ occurs in } w \}$$

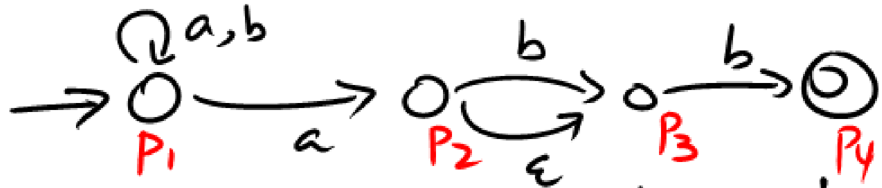


DFA $O(2^{|\Sigma|})$
NFA $O(|\Sigma|)$

A nondeterministic finite automaton (NFA) is a tuple $(Q, \Sigma, \delta, q_0, F)$

- Q is a finite set ("states")
- Σ is a finite set ("alphabet")
- $q_0 \in Q$ ("initial state")
- $F \subseteq Q$ ("final states")
- $\delta : Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$



$$L = \{ wab, wabb \mid w \in \{a,b\}^* \}$$

$$(a+b)^* . (ab + abb)$$

$$(a+b)^* . a . (b+\varepsilon) . b$$

NFA $M = (\{P_1, P_2, P_3, P_4\}, \{a,b\}, \delta, P_1, \{P_4\})$

δ	P_1	P_2	P_3	P_4
a	$\{P_1, P_2\}$	ϕ	ϕ	ϕ
b	$\{P_1\}$	$\{P_3\}$	$\{P_4\}$	ϕ
ε	ϕ	$\{P_3\}$	ϕ	ϕ

Formal notion of acceptance (without ϵ-transitions!)

Let $A = (Q, \Sigma, \delta, q_0, F)$ be an NFA.

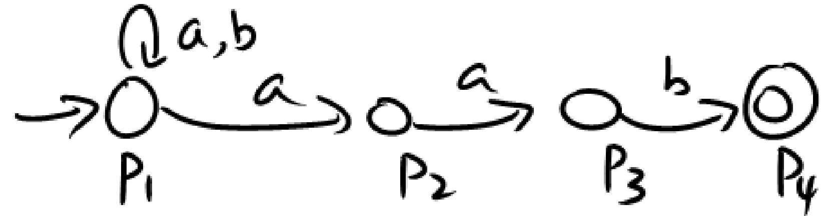
A accepts $w = a_1 a_2 \ldots a_k$

if there is a sequence of states $r_0, r_1, \ldots r_k$ such that

1) $r_0 = q_0$
2) $r_k \in F$
3) $r_{i+1} \in \delta(r_i, a_{i+1})$ for every $i = 0, \ldots k-1$

Note: In a DFA, $r_{i+1} = \delta(r_i, a_{i+1})$

M:



Why is aaab in L(M)?

Because P_1, P_1, P_2, P_3, P_4

P_1 : initial ✓

P_4 : final ✓

$P_1 \in \delta(P_1, a)$

$P_2 \in \delta(P_1, a)$

$P_3 \in \delta(P_2, a)$

$P_4 \in \delta(P_3, b)$