Context-free languages

- Non-closure under complement
- Closure under homomorphisms
- CYK algritum
- Recursive automata.

CFLs are closed under:

$\cup$, $\cdot$, $*$, homomorphism*

(also inv. homomorphisms).

CFLs are <u>not</u> closed under

$\cap$, complement*

* - this class.

CFLs are not closed under complement

$$L = \{a^n b^n c^n \mid n \in \mathbb{N}\}$$ is not a CFL

$$\bar{L} = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\} \cup \overline{a^* b^* c^*}$$

$L' = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$

CFL

L' is a CFL: $L' = L_1 \cup L_2$

$L_1 = \{a^i b^j c^k \mid i \neq j\}$

$L_2 = \{a^i b^j c^k \mid j \neq k\}$

CFLs

L_1 is CFL:

$$
\begin{aligned}
S &\rightarrow AC \\
A &\rightarrow aAb \mid aX \mid bY \\
X &\rightarrow aX \mid \epsilon \\
Y &\rightarrow bY \mid \epsilon \\
C &\rightarrow cC \mid \epsilon
\end{aligned}
$$

$L' = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$
is a CFL.

$\overline{L'} \cap a^* b^* c^* = \{a^n b^n c^n \mid n \in \mathbb{N}\}$

If CFLs are closed under complement,
$\overline{L}$, would be a CFL,
and hence $\overline{L'} \cap a^* b^* c^*$ would be a CFL,
which is a contradiction.

So CFLs are not closed under complement.

CFLs are closed under hompmorphisms

$h : \Sigma \to \Pi^*$

$h : \Sigma^* \to \Pi^*$

$h(a_0 a_1 \ldots a_n) = h(a_0) h(a_1) \ldots h(a_n)$

If L is a CFL, $h(L)$ is a CFL.

Given a grammar G for L.

Construct grammar G' for $h(L)$ by replacing each $a \in \Sigma$ with $h(a)$ in every rule of G.

Membership

If G is in CNF,
then we can check if G derives w
by checking all derivations of
$2n-1$ steps $(n = |w|)$

Exponential! c^{2n-1}

CYK algorithm (Cocke, Schwartz, Younger, Kasami)

a b a a b b a

There are $O(n^2)$ segments.

which vars can generate aabb?

$S \rightarrow AB \mid BC$

$A \rightarrow BA \mid a$

$B \rightarrow CC \mid b$

$C \rightarrow AB \mid a$

n^2 matrix elements.

$O(n)$ time to process an element of the matrix

$O(n^3)$-time algm

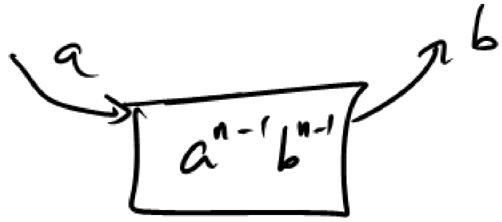
length of substring ↓ / start index →	b 1	a 2	a 3	b 4	a 5
1	B	A, C	A, C	B	A, C
2	A, S	B	S, C	A, S	x
3	ϕ	B	B	x	x
4	ϕ	S, C, A	x	x	x
5	S, A, C	x	x	x	x

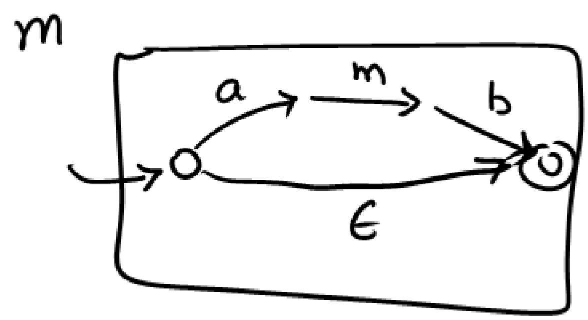
Recursive Automata.

Finite automata – pgm with finite memory.

Recursive automaton – recursive pgm with finite memory.

$$L = \{ a^n b^n \mid n \in \mathbb{N} \}$$



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m() {
either {
  generate a;
  call m;
  generate b;
}
or { generate ε }
}
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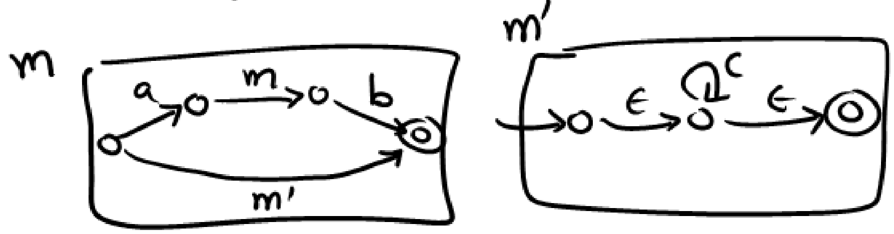
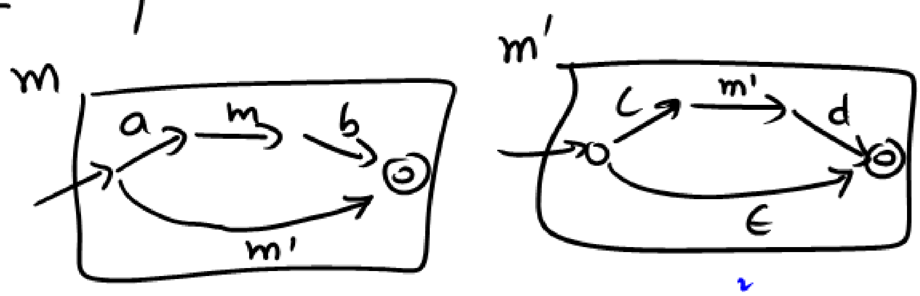
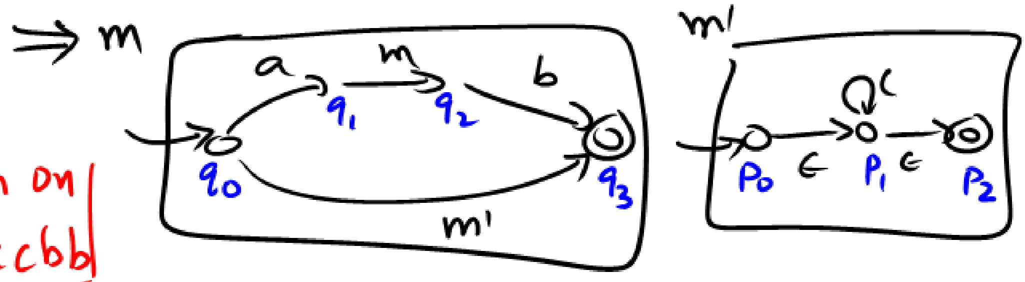
$$L = \{ a^n c^* b^n \mid n \in \mathbb{N}_0 \}.$$



$$L = \{ a^n c^m d^m b^n \mid n, m \in \mathbb{N}_0 \}$$





Run on aacbb

$$(q_0, \sqcup) \xrightarrow{a} (q_1, \sqcup) \xrightarrow{m} \left(q_0, \boxed{q_2}\right) \xrightarrow{a} \left(q_1, \boxed{q_2}\right)$$

$$\xrightarrow{m} \left(q_0, \boxed{\begin{array}{c} q_2 \\ q_2 \end{array}}\right) \xrightarrow{m'} \left(p_0, \boxed{\begin{array}{c} q_3 \\ q_2 \\ q_2 \end{array}}\right) \xrightarrow{\epsilon} \left(p_1, \boxed{\begin{array}{c} q_3 \\ q_2 \\ q_2 \end{array}}\right)$$

$$\xrightarrow{c} \left(p_1, \boxed{\begin{array}{c} q_3 \\ q_2 \\ q_2 \end{array}}\right) \xrightarrow{\epsilon} \left(p_2, \boxed{\begin{array}{c} q_3 \\ q_2 \\ q_2 \end{array}}\right) \xrightarrow{pop} \left(q_3, \boxed{\begin{array}{c} q_2 \\ q_2 \end{array}}\right)$$

$$\xrightarrow{pop} \left(q_2, \boxed{q_2}\right) \xrightarrow{b} \left(q_3, \boxed{q_2}\right) \xrightarrow{pop} (q_2, \sqcup)$$

$$\xrightarrow{b} (q_3, \sqcup) \quad \boxed{\text{END}}$$