

Discussion 1: Review

20 January 2009

Purpose: Most of this material is review from CS 173, though students may have forgotten some of it, especially details of notation. The exceptions are the sections on strings and graph induction.

Before you start, introduce yourself. Promise office hours will be posted very soon and encourage them to come.

1 Homework guidelines

- Put each problem on a different sheet of paper, we grade them separately.
- Put your name on each sheet of paper you submit.
- Put the section number on every sheet of paper you submit.
- Homeworks will usually be due on Thursday at 12:30.
- Homeworks are available from the class webpage.

Do not forget to register with the news server and start reading the news-groups!

2 Numbers

What are Z , N (no zero), N_0 , (mention quickly Q and \mathbb{R}).

3 Divisibility

What does it mean for x to divide q ? Namely, there exists integer n such that $q = xn$. As such, every number n is divisible by 1 and n (i.e., itself).

An integer number is *even* if it can be divided by 2. A number which is not even, is *odd*.

Question 3.1 *Is zero even?*

*Well, $0 = 2 * 0$ and as such, 0 is divisible by two.*

Meaning of $p = q \bmod k$. Examples... (i.e., $121 = 73 \bmod 3$).

4 $\sqrt{2}$ is not rational

A number x is rational if it can be written as the ratio of two integers $x = \alpha/\beta$. The fraction α/β is *irreducible* if α and β do not have any common divisors (except 1, of course). Note that a rational number has a unique way to be written in irreducible.

Lemma 4.1 *An integer k is even if and only if k^2 is even.*

Proof: If k is even, then it can be written as $k = 2u$, where u is an integer. As such, $k^2 = (2u)^2 = 4u^2$ is even, since it can be divided by 2. As for the other possibility, if $k = 2u + 1$, then

$$u^2 = (2k + 1)^2 = 4k^2 + 2 * 2k + 1 = 2(2k^2 + 2k) + 1,$$

is odd (since it is the sum of an even number and an odd number), implying the claim. ■

Theorem 4.2 *The number $\sqrt{2}$ is not rational (i.e., $\sqrt{2}$ is an irrational number).*

Proof: Assume for the sake of contradiction that $\sqrt{2}$ is rational, and it can be written as the irreducible ratio $\sqrt{2} = \alpha/\beta$.

Let us square both sides of this equation. We get that

$$2 = \frac{\alpha^2}{\beta^2}.$$

That is the number 2 is a divisor of α^2 . Namely, α^2 is an even number. But then, α must be an even number. So, let $\alpha = 2a$. We have that

$$2 = \frac{(2a)^2}{\beta^2}. \Rightarrow 2\beta^2 = (2a)^2 = 4a^2.$$

As such,

$$\beta^2 = 2a^2.$$

Namely, β^2 is even, which implies, again that β is even. As such, let us write $\beta = 2b$, where b is an integer. We thus have that

$$\sqrt{2} = \frac{\alpha}{\beta} = \frac{2a}{2b} = \frac{a}{b}.$$

Namely, we started from a rational number in irreducible form (i.e., α/β) and we reduced it further to a/b . But this is impossible. A contradiction. Our assumption that $\sqrt{2}$ is a rational number is false. We conclude that $\sqrt{2}$ is irrational. ■

5 Review of set theory

Sets, set building notation (just show an example), subset, proper subset, empty set, Venn diagram

Tuples, cross product of two sets.

Sets of sets, power set, sets can contain a mixture of stuff like $\{a, b, \{a, b\}\}$.

Demorgan's Laws

Cardinality of a set

6 Strings

strings, empty string, sets of strings

substring, prefix, suffix, string concatenation

What happens when you concatenate the empty string with another string?

Suppose w is a string. Is the empty string a substring of w ? (Yes!)

7 Recursive definition

Recursive definition (concrete example).

Let U be the set that contains the point $(0, 0)$. Also:

- if $(x, y) \in U$, then $(x + 1, y) \in U$.
- if $(x, y) \in U$, then $(x, y + 1) \in U$

Q: What is U ?

A: Clearly, $(0, 1) \in U$, $(0, 2) \in U, \dots (0, k) \in U$ (for any k).

As such, $(1, k) \in U$ (for any k)

As such, $(2, k) \in U$ (for any k)

As such, $(j, k) \in U$ (for any $j > 0, k > 0$).

Note that $U = N_0 \times N_0$.

(A more complicated example of this is in the homework.)

8 Induction example

The following wasn't done in some (all?) of the sections, but you may still find it useful to read.

Consider a graph $G = (V, E)$. The graph G is *connected*, if there is a path between any two vertices of G . A graph is *acyclic* if does not contain any cycle in it. A *leaf* in G is a node that has exactly one edge adjacent to it.

Lemma 8.1 *A connected acyclic graph G over $n > 1$ vertices, must contain a leaf.*

Proof: Indeed, start from a vertex $v \in V(G)$, if it is a leaf we are done, otherwise, there must be an edge $e = vu$ of G adjacent to v (since G is connected), and travel on this edge to u . Mark the edge e as used. Repeat this process of "walking" in the graph till you reach a leaf (and then you stop), where the walk can not use an edge that was used before.

If this walk process reached a leaf then we are done. The other possibility is that the walk never ends. But then, we must visit some vertex we already visited a second time (since the graph is finite). But that would imply that G contains a cycle. But that is impossible, since G is acyclic (i.e., it does not contain cycles). ■

Claim 8.2 *A connected acyclic graph G over n vertices has exactly $n - 1$ edges.*

Proof: The proof is by induction on n .

The base of the induction is $n = 2$. Here we have two vertices, and since its connected, this graph must have the edge connecting the two vertices, implying the claim.

As for $n > 2$, we know by the above lemma that G contains a leaf, and let w denote this leaf.

Consider the graph H formed by G by removing from it the vertex w and the single edge e' attached to it. Clearly the graph H is connected and acyclic, and it has $n - 1$ vertices. By the induction hypothesis, the graph H has $n - 2$ edges. But then, the graph G has all the edges of H plus one (i.e., e'). Namely, G has $(n - 2) + 1 = n - 1$ edges, as claimed. ■