Problem Set 9
CS 373: Theory of Computation

Assigned: November 15, 2013   Due on: November 21, 2013

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lecture 19, 20, and 21.

Problem 1. [Category: Comprehension+Proof] For strings $u, v \in \Sigma^*$, we will say $u < v$ to denote that u is less than v in the lexicographic order. An enumerator N is said to enumerate strings in lexicographic order iff for any strings $u, v \in \mathbf{E}(N)$, if $u < v$ then N prints u before v. In this problem, you are required to prove that a language is decidable iff some enumerator enumerates the language in lexicographic order.

1. Let M be a Turing machine that decides the language L. Show that there is enumerator N such that $\mathbf{E}(N) = L$ and N enumerates the words in L in lexicographic order. **[5 points]**
2. Let N be an enumerator that enumerates strings in lexicographic order. If $\mathbf{E}(N)$ is finite then $\mathbf{E}(N)$ is regular and, therefore, decidable. Prove that if $\mathbf{E}(N)$ is infinite then there is a Turing machine M that decides $\mathbf{E}(N)$. **[5 points]**

Problem 2. [Category: Comprehension+Design] Show that

$$\mathsf{Inf}_{\mathsf{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG such that } \mathbf{L}(G) \text{ is infinite}\}$$

is decidable by outlining an algorithm that decides this problem; you need not prove that your algorithm is correct. *Hint:* You may find it useful to look at the solution for problem 1 in Discussion 12 (or problem 4.10 in the textbook) and think about the pumping lemma for CFGs. **[10 points]**

Problem 3. [Category: Comprehension+Design+Proof] Disjoint languages A and B are said to be *recursively separable* if there is a decidable language L such that $A \subseteq L$ and $B \subseteq \overline{L}$. Prove that if A and B disjoint languages such that $\overline{A}$ and $\overline{B}$ are recursively enumerable then A and B are recursively separable. **[10 points]**