Problem Set 4
CS 373: Theory of Computation

Assigned: September 19, 2013 Due on: September 26, 2013

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website. Solutions not following these guidelines will not be graded.

Recommended Reading: Lectures 7, and 8.

Problem 1. [Category: Comprehension+Design] Consider the language $L = \overline{\mathbf{L}((abb^*)^*)}$.

1. Construct a DFA recognizing L. You need not prove that your construction is correct. If you construct it using the algorithms described in class then you should show all your steps. If you construct the automaton directly then you should explain the intuition behind your construction clearly. **[5 points]**
2. Construct a regular expression for the language L. Again you don't need to prove your regular expression to be correct, but you should show all the steps in the construction. **[5 points]**

Problem 2. [Category: Comprehension+Design+Proof] A string $x \in \Sigma^*$ is said to be a *prefix* of a string $y \in \Sigma^*$ if there is a string z such that $xz = y$. A string x is a *proper prefix* of y if x is a prefix of y and $x \neq y$. For a language $A \subseteq \Sigma^*$ define

$$\text{NOEXTEND}(A) = \{w \in \Sigma^* \mid w \text{ is not the proper prefix of any string in } A\}$$

1. Taking $A = \{\epsilon, 01, 10, 1001\}$, what is NOEXTEND($A$)? **[1 points]**
2. Taking $A = \mathbf{L}(0^*110^*)$, what is NOEXTEND(A)? **[1 points]**
3. Prove that if A is regular then NOEXTEND(A) is regular. You can either construct a DFA/NFA/regular expression for NOEXTEND(A) (and then you don't have to prove that your construction is correct) or use previously established closure properties to prove this result. **[8 points]**

Problem 3. [Category: Comprehension+Design+Proof] For a language $A \subseteq \Sigma^*$ define

$$\text{left}(A) = \{w \in \Sigma^* \mid ww^R \in A\}$$

where w^R denotes the reverse of w.

1. Taking $A = \{\epsilon, 01, 10, 1001\}$, what is left($A$)? **[1 points]**
2. Taking $A = \mathbf{L}(0^*110^*)$, what is left(A)? **[1 points]**
3. Prove that if A is regular then left(A) is regular. You can either construct a DFA/NFA/regular expression for left(A) (and then you don't have to prove that your construction is correct) or use previously established closure properties to prove this result. *Hint:* Look at the construction of halving a language in lecture 4. **[8 points]**