Problem Set 2
CS 373: Theory of Computation

Assigned: September 5, 2013   Due on: September 12, 2013

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website. Solutions not following these guidelines will not be graded.

Recommended Reading: Lectures 2, 3 and 4.

Problem 1. [Category: Design+Proof] Let $A_k \subseteq \{a,b\}^*$ be the collection of strings w where there is a position i in w such that the symbol at position i (in w) is a, and the symbol at position $i+k$ is b. For example, consider A_2 (when $k=2$). $baab \in A_2$ because the second position ($i=2$) has an a and the fourth position has a b. On the other hand, $bb \notin A_2$ (because there are no as) and $aba \notin A_2$ (because none of the as are followed by a b 2 positions away).

1. Design a DFA for language A_k. Your formal description (by listing states, transitions, etc. and not "drawing the DFA") will depend on the parameter k but should work no matter what k is; see lecture 2, last page for such an example. **[5 points]**
2. Prove that your DFA is correct when $k=2$. **[5 points]**

Problem 2. [Category: Comprehension] Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ define the following function $\rho : \Sigma^* \to 2^{Q \times Q}$ inductively. (Intuitively, ρ maps a string to a binary relation on states Q.)

$$\rho(w) = \begin{cases} \{(q_1,q_2) \mid q_2 \in \hat{\delta}_M(q_1,\epsilon)\} & \text{if } w = \epsilon \\ \{(q_1,q_2) \mid \text{exists } q' \in Q.\ q_2 \in \hat{\delta}_M(q',a) \text{ and } (q_1,q') \in \rho(u)\} & \text{if } w = ua \end{cases}$$

where $u \in \Sigma^*$ and $a \in \Sigma$. Answer the following questions about ρ and the NFA M_0 (over the alphabet $\{0,1\}$) shown below.

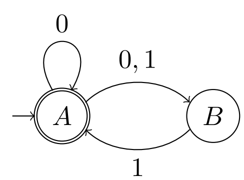

Figure 1: NFA M_0 for Problem 2

1. What is $\rho(\epsilon)$, $\rho(010)$, $\rho(101)$, and $\rho(110)$? **[4 points]**
2. Give an english/mathematical description of what ρ is for a general NFA. **[1 points]**
3. For a NFA M, define $\mathbf{L}'(M) = \{w \in \Sigma^* \mid \exists q \in F.\ (q_0,q) \in \rho(w)\}$. For each of the following answer whether the belong to $\mathbf{L}'(M_0)$: 101, 110? **[2 points]**

4. What is $\mathbf{L}'(M_0)$? **[2 points]**

5. For a general NFA M, what is the relationship between $\mathbf{L}(M)$ and $\mathbf{L}'(M)$? (Answer which of the following best describes the relationship: $\mathbf{L}(M) = \mathbf{L}'(M)$, $\mathbf{L}(M) \subseteq \mathbf{L}'(M)$ or $\mathbf{L}'(M) \subseteq \mathbf{L}(M)$.) **[1 points]**

Problem 3. [Category: Design+Proof] Consider the language $A_2 \subseteq \{a, b\}^*$, from problem 1, which was defined to be the collection of strings w where there is a position i in w such that the symbol at position i (in w) is a, and the symbol at position $i + 2$ is b.

1. Design an NFA for language A_2 that has at most 4 states. You need not prove that your construction is correct, but the intuition behind your solution should be clear and understandable. **[5 points]**

2. Prove that any DFA recognizing A_2 has at least 5 states. **[5 points]**