Problem Set 9
CS 373: Theory of Computation

Assigned: November 27, 2012 Due on: December 4, 2012

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lectures 21 through 23.

Problem 1. [Category: Proof] Show that every infinite recursively enumerable language has an infinite decidable subset. That is, if A is infinite and r.e. then there is B such that $B \subseteq A$ and B is decidable. *Hint:* Recall that in discussion 13 (November 27–28) you saw the following result: L is decidable iff some enumerator enumerates the strings in L in lexicographic order.

Problem 2. [Category: Proof] Let C be a language. Prove that C is recursively enumerable iff a decidable language D exists such that $C = \{x \mid \exists y.\ \langle x, y\rangle \in D\}$.

Problem 3. [Category: Proof] Consider $\text{Inf} = \{\langle M\rangle \mid M \text{ is a TM and } \mathbf{L}(M) \text{ is infinite}\}$. Using reductions, prove that Inf is not recursively enumerable. *Hint:* Reduce a known non-r.e. language like $\overline{A_{\text{TM}}}$ or L_d to Inf.