Problem Set 5
CS 373: Theory of Computation

Assigned: October 11, 2012   Due on: October 18, 2012

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lectures 11 and 12.

Problem 1. [Category: Comprehension+Design] Let $L = \mathbf{L}(1^*0(00 \cup 01 \cup 1)(0 \cup 1)^*)$.

1. List all the equivalence classes of $\equiv_L$. Prove that your answer is correct. **[5 points]**

2. Draw the minimum state DFA M^L accepting L. **[5 points]**

Problem 2. [Category: Comprehension] Consider the following grammar over the terminals $\{0, \#\}$ and start symbol S.

$$S \to TT \mid U$$
$$T \to 0T \mid T0 \mid \#$$
$$U \to 0U00 \mid \#$$

1. For each of the following strings, answer whether or not they belong to the language defined by the grammar: $00\#0\#00$, $0\#0$, $000\#000000$. If they do, give a derivation and parse tree for the string. If not, provide a short, informal justification for why the string cannot be generated. **[6 points]**

2. What is the language of the grammar? You need not prove your answer, but you should provide a short informal justification for your answer. **[2 points]**

3. Is the language of the grammar regular? Prove your answer. **[2 points]**

Problem 3. [Category: Design+Proof] Design a context-free grammar for the language $L = \{a^i b^j \mid 2i \leq j \leq 3i, \ i, j \in \mathbb{N}\}$. Provide a formal proof that your construction is correct. *Hint:* Build a grammar for the case when $j = 2i$ and $j = 3i$, and think of a way to fuse the two together. **[10 points]**