Problem Set 4
CS 373: Theory of Computation

Assigned: September 20, 2012   Due on: September 27, 2012

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lectures 7 and 8.

Problem 1. [Category: Proof] Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be an NFA recognizing language L. Recall that in lecture 7 we gave the following construction of an NFA recognizing L^*: Take $N = (Q, \Sigma, \delta, q_0, F)$ where

- $Q = Q_1 \cup \{q_0\}$, where $q_0 \notin Q_1$
- $F = F_1 \cup \{q_0\}$
- δ is defined as follows

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \in (Q_1 \setminus F_1) \text{ or } a \neq \epsilon \\ \delta_1(q,a) \cup \{q_1\} & \text{if } q \in F_1 \text{ and } a = \epsilon \\ \{q_1\} & \text{if } q = q_0 \text{ and } a = \epsilon \\ \emptyset & \text{otherwise} \end{cases}$$

Prove that N does indeed recognize L^*, i.e., $\mathbf{L}(N) = (\mathbf{L}(N_1))^* = L^*$. **[10 points]**

Problem 2. [Category: Proof] Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be DFAs recognizing L_1 and L_2, respectively. Recall that in lecture 8, we gave the following construction of a DFA recognizing the intersection of L_1 and L_2: $M = (Q, \Sigma, \delta, q_0, F)$ defined as follows

- $Q = Q_1 \times Q_2$
- $q_0 = \langle q_1, q_2 \rangle$
- $\delta(\langle p_1, p_2 \rangle, a) = \langle \delta_1(p_1, a), \delta_2(p_2, a) \rangle$
- $F = F_1 \times F_2$

Prove that $\mathbf{L}(M) = \mathbf{L}(M_1) \cap \mathbf{L}(M_2)$. **[10 points]**

Problem 3. [Category: Comprehension+Design] For languages A and B (over alphabet Σ), let PerfectShuffle(A, B) be the language

$$\{w \in \Sigma^* \mid w = a_1 b_1 a_2 b_2 \cdots a_n b_n, \text{ where, for each } i, \ a_i, b_i \in \Sigma \text{ and } a_1 a_2 \cdots a_n \in A \text{ and } b_1 b_2 \cdots b_n \in B\}$$

1. Let $\Sigma = \{0, 1\}$. Let $A = \mathbf{L}(0^*)$ and $B = \mathbf{L}(1^*)$. What is PerfectShuffle(A, B)? **[1 points]**

2. Let $\Sigma = \{0\}$. Let $C = \{w \in \Sigma^* \mid |w| \text{ is even}\}$ and $D = \{w \in \Sigma^* \mid |w| \text{ is prime}\}$. What is PerfectShuffle$(C, D)$? **[1 points]**

3. Prove that regular languages are closed under the perfect shuffle operator, i.e., if A and B are regular then PerfectShuffle(A, B) is regular. If you prove regularity of PerfectShuffle(A, B) by constructing a DFA/NFA/regular expression then you need not prove the correctness of your construction; you should, however, clearly explain the intuition behind your constrauction. [**8 points**].