# Problem Set 3
## CS 373: Theory of Computation

Assigned: September 13, 2012    Due on: September 20, 2012

**Instructions:** This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

**Recommended Reading:** Lectures 3,4,5 and 6.

**Problem 1**. [Category: Design+Proof] Let $\Sigma = \{0, 1\}$. Let $W_k = \{w_1 w_2 \mid w_1, w_2 \in \Sigma^k \text{ and } w_1 \neq w_2\}$

1. Prove that, for every $k$, any DFA recognizing $W_k$ must have at least $2^k$ states.
2. Construct an NFA that recognizes $W_k$ with $O(k^2)$ states. You need not prove that your construction is correct, but you should clearly explain the intuition behind your design.

**Problem 2**. [Category: Comprehension+Design]

1. Describe the language of the following regular expressions. A clear, crisp one-level interpretable English description is acceptable, like "This is the set of all binary strings with at least three 0s and at most hundred 1s", or like "$\{0^n(10)^m \mid n$ and $m$ are integers$\}$".A vague, recursive or multi-level-interpretable description is not, like "This is a set of binary strings that starts and ends in 1, and the rest of the string starts and ends in 0, and the remainder of the string is a smaller string of the same form!" or "This is a set of strings like 010, 00100, 0001000, and so on!". You need not prove the correctness of your answer.
   (a) $(0^* \cup 0 \cup 1^*)^*$ **[1 points]**
   (b) $1(00^*1)^*0^*$ **[2 points]**
   (c) $1^*(0 \cup 111^*)^*1^*$ **[2 points]**
2. Give regular expressions that accurately describe the following languages. You need not prove the correctness of your answer.
   (a) All binary strings such that if it starts with 0 it has odd length and if it starts with 1 it has even length. **[1 points]**
   (b) All binary strings such that in every prefix, the number of 0s and 1s differ by at most 1. **[2 points]**
   (c) All binary strings such that every pair of consecutive 0s appears before any pair of consecutive 1s. **[2 points]**

**Problem 3**. [Category: Design] Let $\text{SUFFIX}(L) = \{y \in \Sigma^* \mid \exists x \in \Sigma^*.\ xy \in L\}$. Let $r, s, r_T, s_T$ be the regular expressions for languages $R, S, \text{SUFFIX}(R)$, and $\text{SUFFIX}(S)$, respectively. Using only these regular expressions and the operations $\cup$, concatenation, and $*$, give the regular expressions for the following languages:

(a) SUFFIX($R \cup S$) **[2 points]**

(b) SUFFIX($RS$) **[2 points]**

(c) SUFFIX($R^*$) **[2 points]**

Using the solutions to (a), (b), and (c), give an inductive algorithm that constructs the regular expression for SUFFIX($L$) from the regular expression for $L$. You need not prove that your algorithm is correct. **[4 points]**