Problem Set 2
CS 373: Theory of Computation

Assigned: September 6, 2012 Due on: September 13, 2012

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lectures 3 and 4.

Problem 1. [Category: Design+Proof] Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

Σ_2 contains all size 2 columns of 0s and 1s. A string of symbols in Σ_2 gives 2 rows of 0s and 1s. Consider each row to be a binary number, where the first symbol is the least significant bit of each binary number. For example, the string

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

represents $0011 = 3$ (first row) and $1001 = 9$ (third row). Let

$$C = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is 3 times the top row}\}$$

For example,

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in C \qquad \text{but} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \notin C$$

1. Design a DFA that recognizes C. **[5 points]**
2. Prove that your construction is correct. **[5 points]**

Problem 2. [Category: Proof] Consider the language $L = \{w \in \{0\}^* \mid |w| \bmod 3 = 0\}$ over the unary alphabet $\{0\}$. Prove that any DFA that recognizes L must have at least 3 states. **[10 points]**

Problem 3. [Category: Comprehension] Consider the following NFA M_0 over the alphabet $\{0, 1\}$.

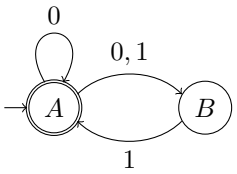

Figure 1: NFA M_0 for Problem 3

1. Describe formally what the following are for automaton M_0: set of states, initial state, final states, and transition function. **[4 points]**

2. What are $\hat{\delta}_{M_0}(A, 010)$, $\hat{\delta}_{M_0}(A, 101)$, $\hat{\delta}_{M_0}(A, 1101)$, and $\hat{\delta}_{M_0}(B, 10)$? **[4 points]**

3. What is $\mathbf{L}(M_0)$? You don't have to prove your answer. **[2 points]**