Problem Set 2  
CS 373: Theory of Computation  
Assigned: September 6, 2012  
Due on: September 13, 2012

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submissions not following these guidelines will not be graded.

Recommended Reading: Lectures 3 and 4.

Problem 1. [Category: Design+Proof] Let

\[ \Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} \]

\( \Sigma_2 \) contains all size 2 columns of 0s and 1s. A string of symbols in \( \Sigma_2 \) gives 2 rows of 0s and 1s. Consider each row to be a binary number, where the first symbol is the least significant bit of each binary number. For example, the string

\[ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

represents 0011 = 3 (first row) and 1001 = 9 (third row). Let

\( C = \{ w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is 3 times the top row} \} \)

For example,

\[ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in C \quad \text{but} \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \not\in C \]

1. Design a DFA that recognizes \( C \). [5 points]
2. Prove that your construction is correct. [5 points]

Problem 2. [Category: Proof] Consider the language \( L = \{ w \in \{0\}^* \mid |w| \mod 3 = 0 \} \) over the unary alphabet \( \{0\} \). Prove that any DFA that recognizes \( L \) must have at least 3 states. [10 points]

Problem 3. [Category: Comprehension] Consider the following NFA \( M_0 \) over the alphabet \( \{0, 1\} \).

![Figure 1: NFA \( M_0 \) for Problem 3](image.png)
1. Describe formally what the following are for automaton $M_0$: set of states, initial state, final states, and transition function. [4 points]

2. What are $\hat{\delta}_{M_0}(A, 010)$, $\hat{\delta}_{M_0}(A, 101)$, $\hat{\delta}_{M_0}(A, 1101)$, and $\hat{\delta}_{M_0}(B, 10)$? [4 points]

3. What is $L(M_0)$? You don’t have to prove your answer. [2 points]