Problem Set 10
CS 373: Theory of Computation

Assigned: December 4, 2012   Due on: December 11, 2012

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lectures 23 through 25.

Problem 1. [Category: Proof] Let $A, B \subseteq \{0,1\}^*$ be r.e. languages such that $A \cup B = \{0,1\}^*$ and $A \cap B \neq \emptyset$. Prove that $A \leq_m (A \cap B)$. **[10 points]**

Problem 2. [Category: Proof] Define a *two-headed finite automaton* (2DFA) to be a deterministic finite automaton that two read-only, bidirectional heads that start at the left-end of the input tape, and can be independently controlled to move in either direction. Thus, in each step, the machine reads the symbols under each head, and based on the current control state, the next control state is determined, along with the direction in which to move each tape head. We assume that at the begining of the computation, the (input) tape contains the string $\rhd w \lhd$, where w is the input string, and $\rhd$ and $\lhd$ are the left and right end-marker that help the machine realize when it has moved beyond the input. As always, a 2DFA accepts a string w if the machine reaches an accepting control state on input w (i.e., when the tape contains $\rhd w \lhd$).

A 2DFA is more powerful that a (regular) DFA. This can be seen from the fact that a 2DFA can recognize the language $\{a^n b^n c^n \mid n \geq 0\}$. This is surprising because a DFA that has only one read-only bidirectional head can only recognize regular languages, since such a machine is a special kind of NITWIT machine (that can never even leave the input portion to write outside).

1. Let $A_{\text{2DFA}} = \{\langle M, w\rangle \mid M \text{ is a 2DFA and } M \text{ accepts } w\}$. Prove that A_{2DFA} is decidable. *Hint:* Think about what the configuration of such a machine would look like, and based on that see if you can bound the number of steps M can take before accepting an input w. **[5 points]**
2. Let $E_{\text{2DFA}} = \{\langle M\rangle \mid M \text{ is a 2DFA and } \mathbf{L}(M) = \emptyset\}$. Prove that E_{2DFA} is undecidable. *Hint:* Recall that an accepting TM computation on input w is a string $c_1\#c_2\#\cdots\#c_n$, where c_1 is the initial conifguration with input w, c_n is an accepting computation, and for all i $c_i \vdash c_{i+1}$. To prove undecidability, reduce $\overline{A_{\text{TM}}}$ to E_{2DFA} as follows: The reduction f on input $\langle M, w\rangle$ produces a code $\langle D\rangle$, where D is 2DFA that accepts all the accepting configurations of M on w. You need not describe the 2DFA precisely; you can use some pseudo-code to show how D can recognize accepting configurations of M. **[5 points]**

Problem 3. [Category: Proof] Recall that P is the collection of languages that can be solved in (deterministic) polynomial time. Prove that P is closed under the following operations:

1. Concatenation **[5 points]**
2. Inverse homomorphic images **[5 points]**