Problem Set 1
CS 373: Theory of Computation

Assigned: August 30, 2012 Due on: September 6, 2012

Instructions: This homework has 3 problems that can be solved in groups of size at most 3. Please follow the homework guidelines given on the class website; submitions not following these guidelines will not be graded.

Recommended Reading: Lectures 1 and 2.

Problem 1. [Category: Comprehension+Proof]

1. Let $A = \{1, 2, 3\}$, $B = \{\emptyset, \{1\}, \{2\}\}$, and $C = \{1, 2, \{1, 2\}\}$. Compute $A \cup B$, $A \cap B$, $B \cap C$, $A \cap C$, $A \times B$, $A \times C$, $C \setminus A$, $C \setminus B$, $A \times B \times C$, and 2^B. Recall that 2^A denotes the *power set* of A, and $A \setminus B$ denotes A *set difference* B. **[5 points]**
2. Prove for any sets A, B, and C, $A \times (B \cup C) = (A \times B) \cup (A \times C)$. **[5 points]**

Problem 2. [Category: Comprehension] Consider the following DFA M_0 over the alphabet $\{0, 1\}$.

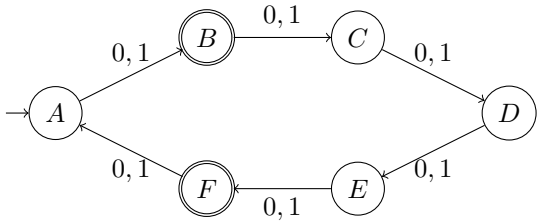

Figure 1: DFA M_0 for Problems 2 and 3

1. Describe formally what the following are for automaton M_0: set of states, initial state, final states, and transition function. **[4 points]**
2. What are $\hat{\delta}_{M_0}(A, \epsilon)$, $\hat{\delta}_{M_0}(A, 1011)$, $\hat{\delta}_{M_0}(B, 101)$, and $\hat{\delta}_{M_0}(C, 10110)$? **[4 points]**
3. What is $\mathbf{L}(M_0)$? **[1 points]**
4. What is the language recognized if we change the initial state to B? What is the language recognized if we change the set of final states to be $\{B\}$ (with initial state A)? **[1 points]**

Problem 3. [Category: Comprehension] Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$ define the following function $\rho : \Sigma^* \to 2^{Q \times Q}$ inductively. (Intuitively, ρ maps a string to a binary relation on states Q.)

$$\rho(w) = \begin{cases} \{(q, q) \mid q \in Q\} & \text{if } w = \epsilon \\ \{(q_1, q_2) \mid \text{exists } q' \in Q.\ q_2 = \delta(q', a) \text{ and } (q_1, q') \in \rho(u)\} & \text{if } w = ua \end{cases}$$

where $u \in \Sigma^*$ and $a \in \Sigma$. Answer the following questions about ρ and the DFA M_0 from problem 2.

1. What is $\rho(\epsilon)$, $\rho(1011)$, $\rho(101)$, and $\rho(10110)$? **[4 points]**

2. Give an english/mathematical description of what ρ is for a general DFA. **[1 points]**

3. For a DFA M, define $\mathbf{L}'(M) = \{w \in \Sigma^* \mid \exists q \in F.\, (q_0, q) \in \rho(w)\}$. For each of the following answer whether the belong to $\mathbf{L}'(M_0)$: 10110, 101? **[2 points]**

4. What is $\mathbf{L}'(M_0)$? **[2 points]**

5. For a general DFA M, what is the relationship between $\mathbf{L}(M)$ and $\mathbf{L}'(M)$? (Answer which of the following best describes the relationship: $\mathbf{L}(M) = \mathbf{L}'(M)$, $\mathbf{L}(M) \subseteq \mathbf{L}'(M)$ or $\mathbf{L}'(M) \subseteq \mathbf{L}(M)$.) **[1 points]**