Recap

• (Ch 11) Learning to classify
  • Nearest neighbors classifier
  • Naïve Bayes classifier
  • Support vector machine (SVM) classifier

Today

• (Ch 11) Learning to classify
  • Decision trees and random forest classifier
  • Comparing classifiers
Decision tree: object classification example

This object classification decision tree can classify into 6 classes.
Training a decision tree: example
Training a decision tree

• Choose a dimension and a split

• Split the training data $D$ into left- and right-child subsets $D_l$ and $D_r$

• Repeat the two steps above recursively on each child

• Stop the recursion based on some conditions

• Label the leaves with class labels
Classifying with a decision tree: example

\[ y > 0.32 \]
\[ x > -0.58 \]
\[ x > 1.06 \]

So ? is classified as \( \bigcirc \)
Choosing a split

An **informative** split makes the subsets more concentrated and **reduces uncertainty** about class labels.
Which split is more informative?
Quantifying uncertainty using entropy

• We can measure uncertainty as the number of bits of information needed to distinguish between classes in a dataset
  • We need $\log_2 2 = 1$ bit to tell apart 2 equally likely classes
  • We need $\log_2 4 = 2$ bits to tell apart 4 equally likely classes

• Entropy is the measure of uncertainty for a general distribution
  • If class $i$ contains a fraction $P(i)$ of the data, we need $\log_2 \frac{1}{P(i)}$ bits for that class
  • The entropy $H(D)$ of a dataset $D$ is the weighted mean across all $c$ classes

$$H(D) = \sum_{i=1}^{c} P(i) \log_2 \frac{1}{P(i)} \text{ bits}$$
Entropy: examples

\[ D \]

\[ P(\omega) = \frac{3}{5} \]
\[ P(\emptyset) = \frac{2}{5} \]

\[ H(D) = \frac{3}{5} \log_2 \frac{5}{3} + \frac{2}{5} \log_2 \frac{5}{2} \]
\[ = 0.971 \text{ bits} \]

\[ D_L \]
\[ P(x) = 1 \]
\[ P(\emptyset) = \frac{1}{2} \]

\[ H(D_L) = \log_2 1 = 0 \text{ bits} \]

\[ D_r \]
\[ P(x) = \frac{1}{3} \]
\[ P(\emptyset) = \frac{2}{3} \]

\[ H(D_r) = \frac{1}{3} \log_2 3 + \frac{2}{3} \log_2 \frac{3}{2} \]
\[ = 0.918 \text{ bits} \]
Information gain of a split

The information gain of a split is the amount it reduces entropy on average

\[ I = H(D) - \left( \frac{N_{D_l}}{N_D} H(D_l) + \frac{N_{D_r}}{N_D} H(D_r) \right) \]

where

- \( N_D \) is the number of items in dataset \( D \)
- \( N_{D_l} \) is the number of items in left-child dataset \( D_l \)
- \( N_{D_r} \) is the number of items in right-child dataset \( D_r \)
Information gain: example

\[ I = H(D) - \left( \frac{24}{60} H(D_L) + \frac{36}{60} H(D_R) \right) = 0.420 \text{ bits} \]
How to choose a dimension and split

• If there are $d$ dimensions, choose approximately $\sqrt{d}$ of them as candidates at random

• For each candidate, find the split that maximizes information gain

• Choose the best overall dimension and split

• Note that splitting can be generalized to categorical features for which there is no natural ordering of the data
When to stop growing the decision tree

• Growing the tree too deep can lead to overfitting to the training data

• Stop recursion on a data subset if any of the following occurs
  • All items in the data subset are in the same class
  • The data subset becomes smaller than a predetermined size
  • A predetermined maximum tree depth has been reached
How to label the leaves of the decision tree

• A leaf will usually have a data subset containing many class labels

• Choose the class that has the most items in the subset

• Alternatively, label the leaf with the number of items it contains in each class for a probabilistic “soft” classification
From decision trees to random forests (RF)

- Decision trees have some drawbacks
  - May not perform well on training data because of simplistic random training
  - May not perform well on test data because of overfitting

- A random forest is a randomly generated ensemble of decision trees that avoids both of the above problems by merging the classifications of the individual trees
Training, evaluation and classification

- Build the random forest by training each decision tree on a new bootstrap replicate of the training data (called a bag)

- Evaluate the random forest by evaluating each decision tree on its out-of-bag items

- Classify by merging the classifications of individual decision trees
  - By simple vote
  - Or by adding soft classifications together and then taking a vote
Considerations in choosing a classifier

• When solving a classification problem, it’s usually a good idea to try several techniques

• Here are some criteria with corresponding strong classifiers
  • Accuracy (SVM, random forests)
  • Training speed (nearest neighbors, naïve Bayes)
  • Classification speed (naïve Bayes, SVM)
  • Performance with small training dataset (naïve Bayes)
  • Interpretability (nearest neighbors, naïve Bayes, SVM)
HW 10 Problem 2

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369 training $\rightarrow$ 50

100 validation $\rightarrow$ choose best $\lambda$

100 test $\rightarrow$ accuracy/mean reporting