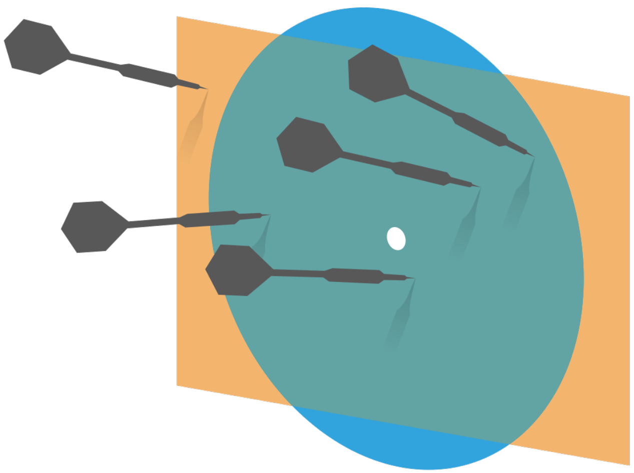
Probability and Statistics for Computer Science



Credit: wikipedia

"Unsupervised learning is arguably more typical of human and animal learning…"--- Kelvin Murphy, former professor at UBC

Hongye Liu, Teaching Assistant Prof, CS361, UIUC, 12.03.2020

Last time

- Curse of dimensions
- Unsupervised learning
- Clustering

Objectives

* Application of Clustering

Cluster center Histogram

* Markov chain (I)

conditional prob.

coming back is Matrix

Q. Is k-means clustering deterministic?

A. Yes

B. No

K-means clustering example: Portugal consumers

- The dataset consists of the annual grocery spending of 440 customers

- Each customer's spending is recorded in 6 features:
  - fresh food, milk, grocery, frozen, detergents/paper, delicatessen

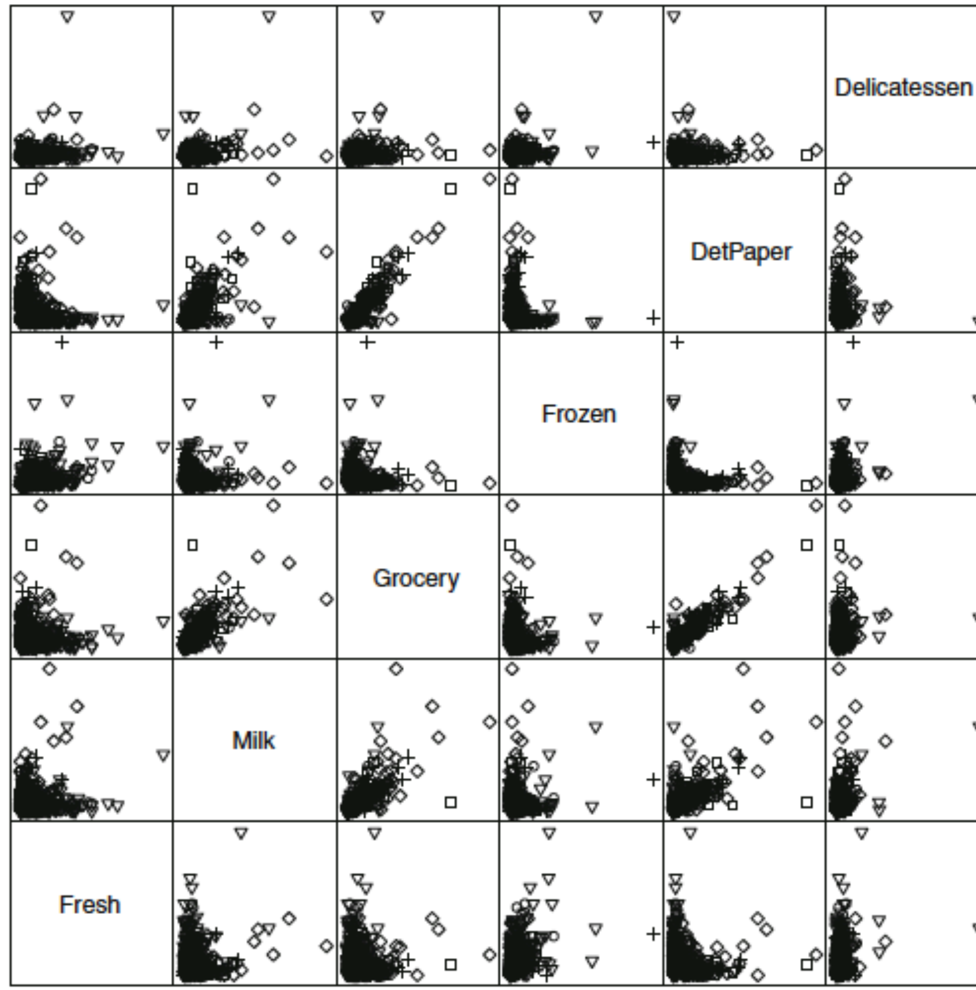
- Each customer is labeled by: 6 labels in total
  - Channel (Channel 1 & 2) (Horeca 298, Retail 142)
  - Region (Region 1, 2 &3) (Lisbon 77, Oporto 47, Other 316)

Lisbon, Portugal

Oporto, Portugal

Visualization of the data

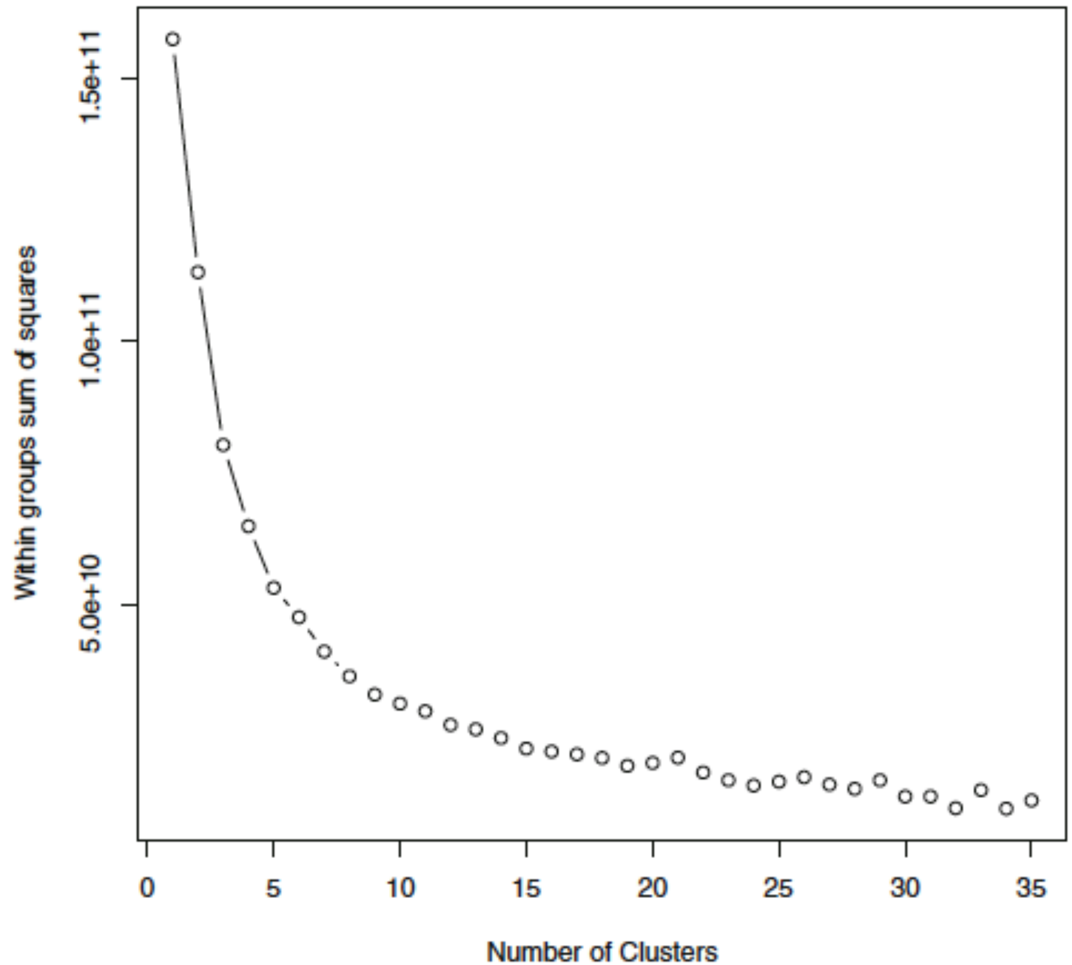
- Visualize the data with scatter plots
- We do see that some features are correlated.
- But overall we do not see significant structure or groups in the data.



Scatter Plot Matrix

Do kmeans and choose k through the cost function

It's good to pick a **k** around the knee:
I choose 6 for it matches the number of labels



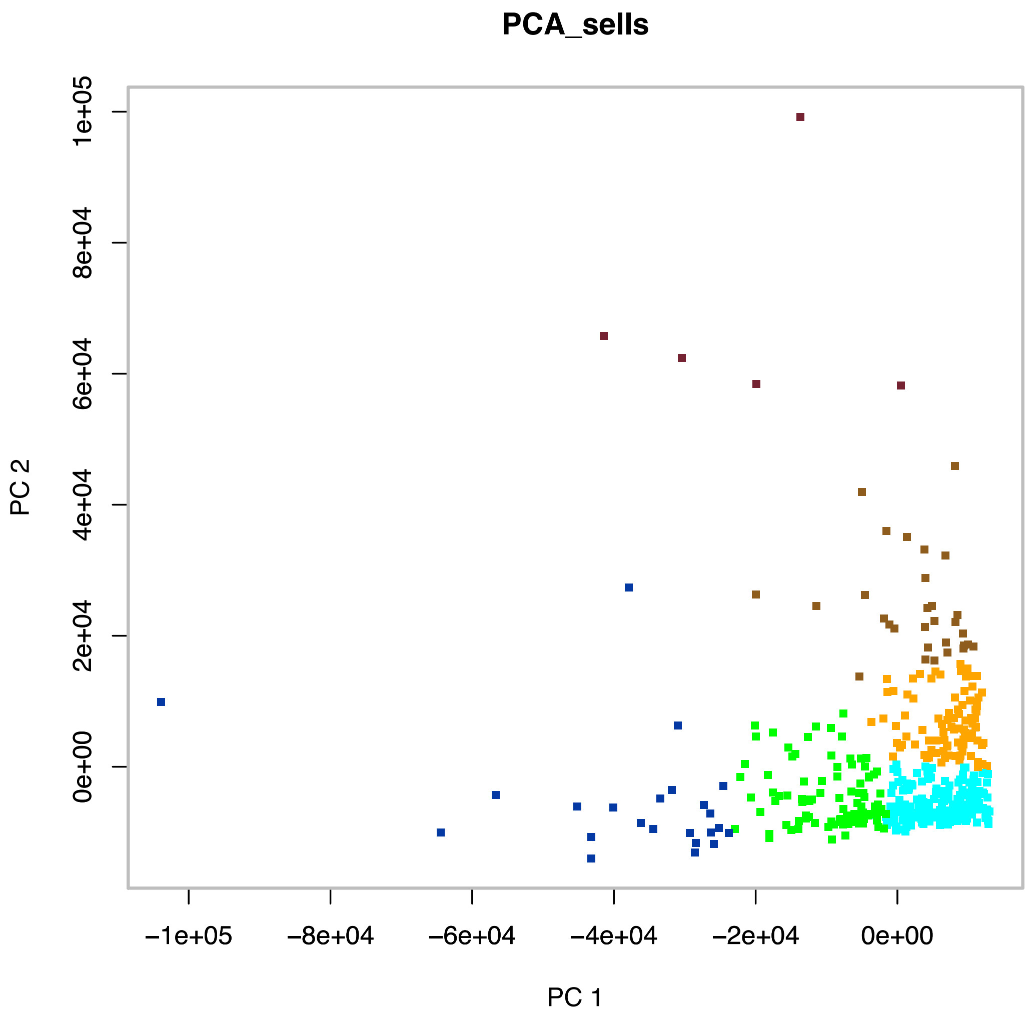
Visualization of the data (PCA)

- PCA does show some separation. **Colors are the clusters**

- Data points show large range of dynamics!

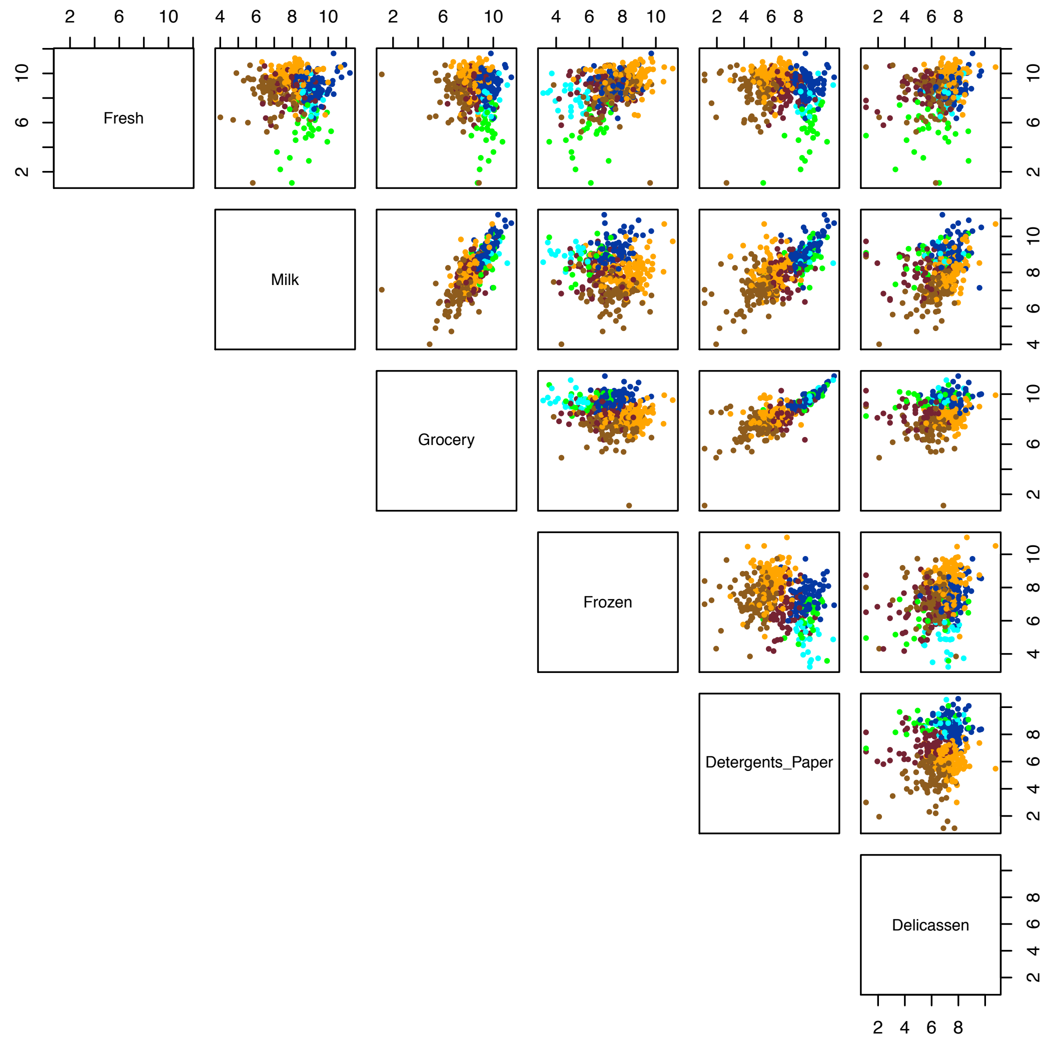
440 customers

each dot is one customer



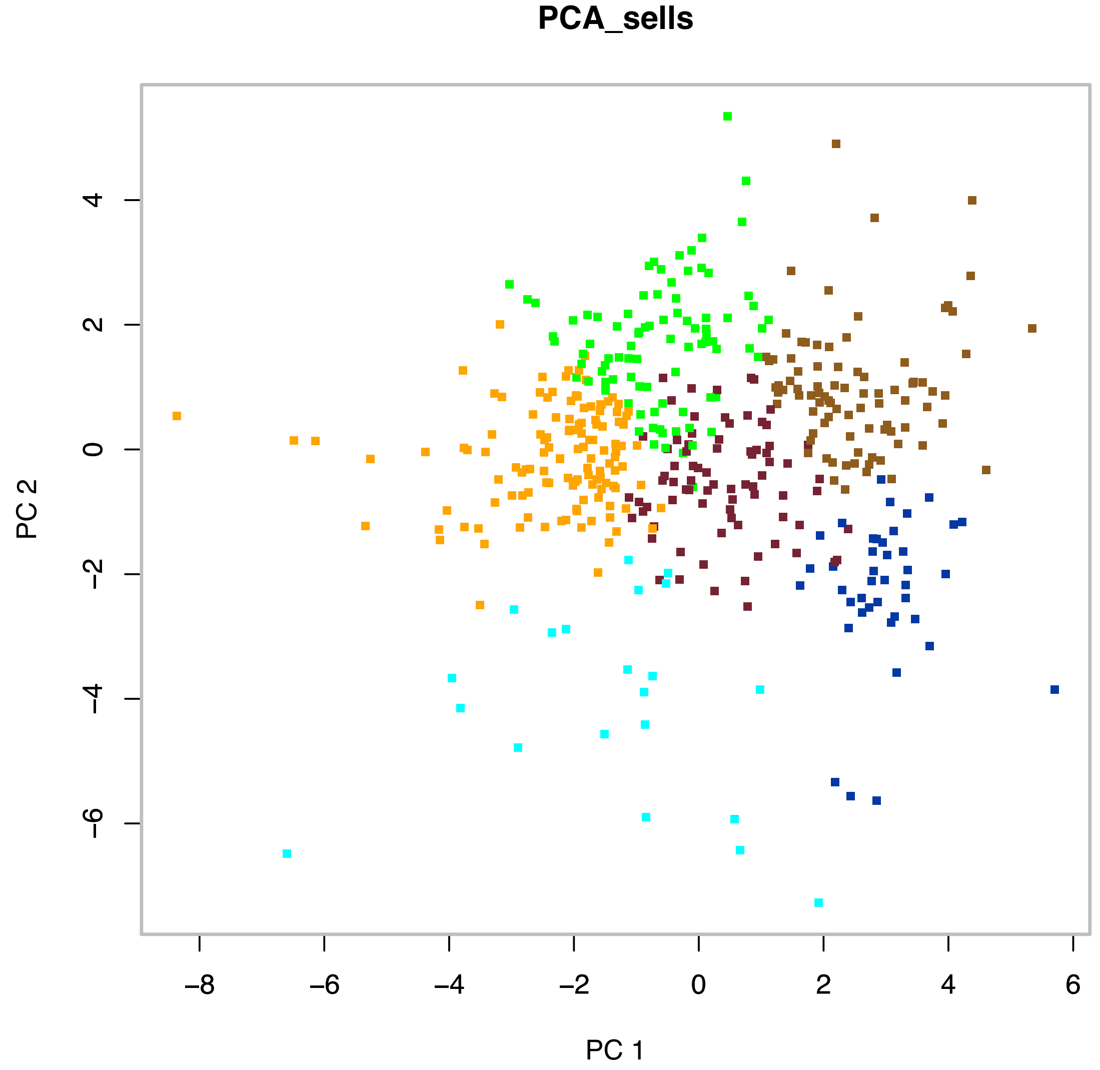
Do log transform of the data

- Log transform the data
- Do scatter plot matrix after the log transform
- Do the kmeans and color the clusters identified by k-means



PCA after log transformation: Clusters

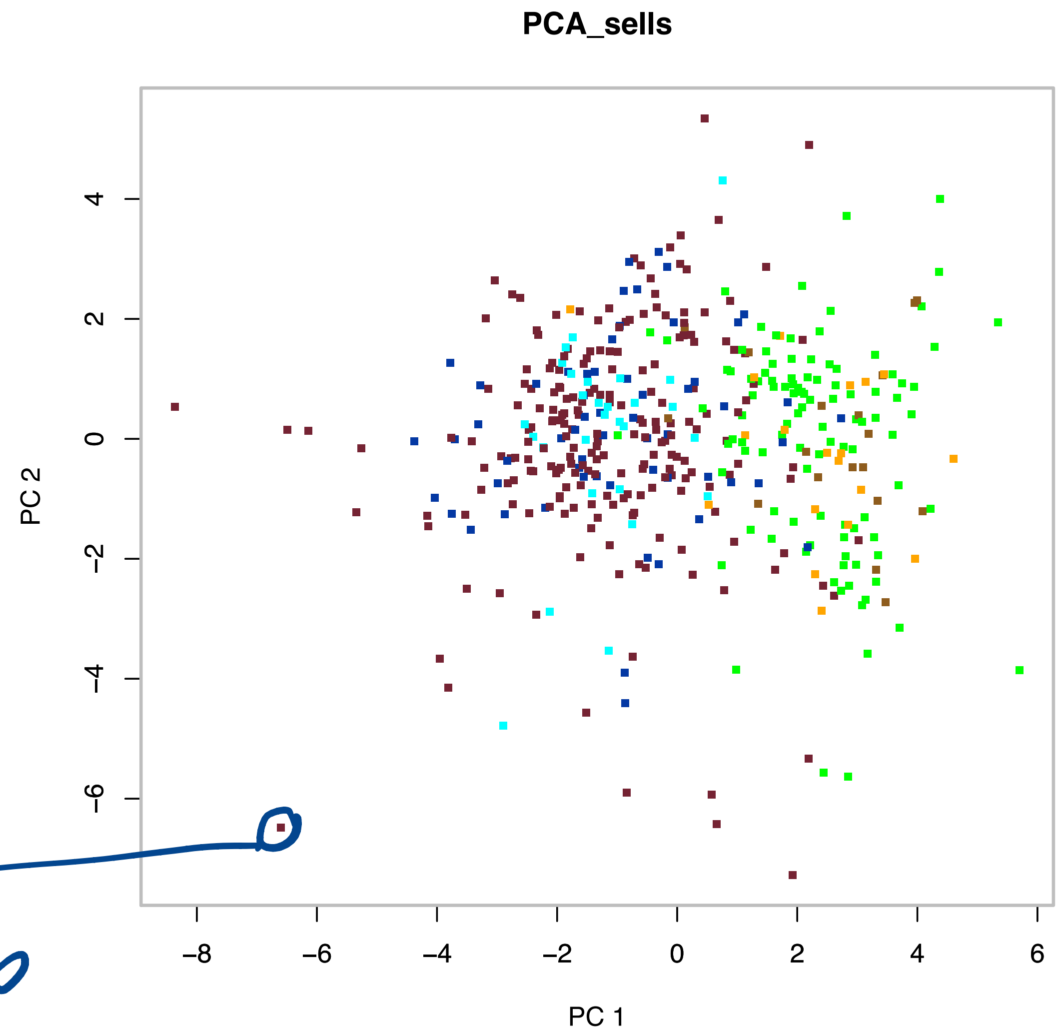
Colors show the **clusters** identified by k-means



PCA after log transformation

Colors show the **Channel-region labels**

What does this tell us?

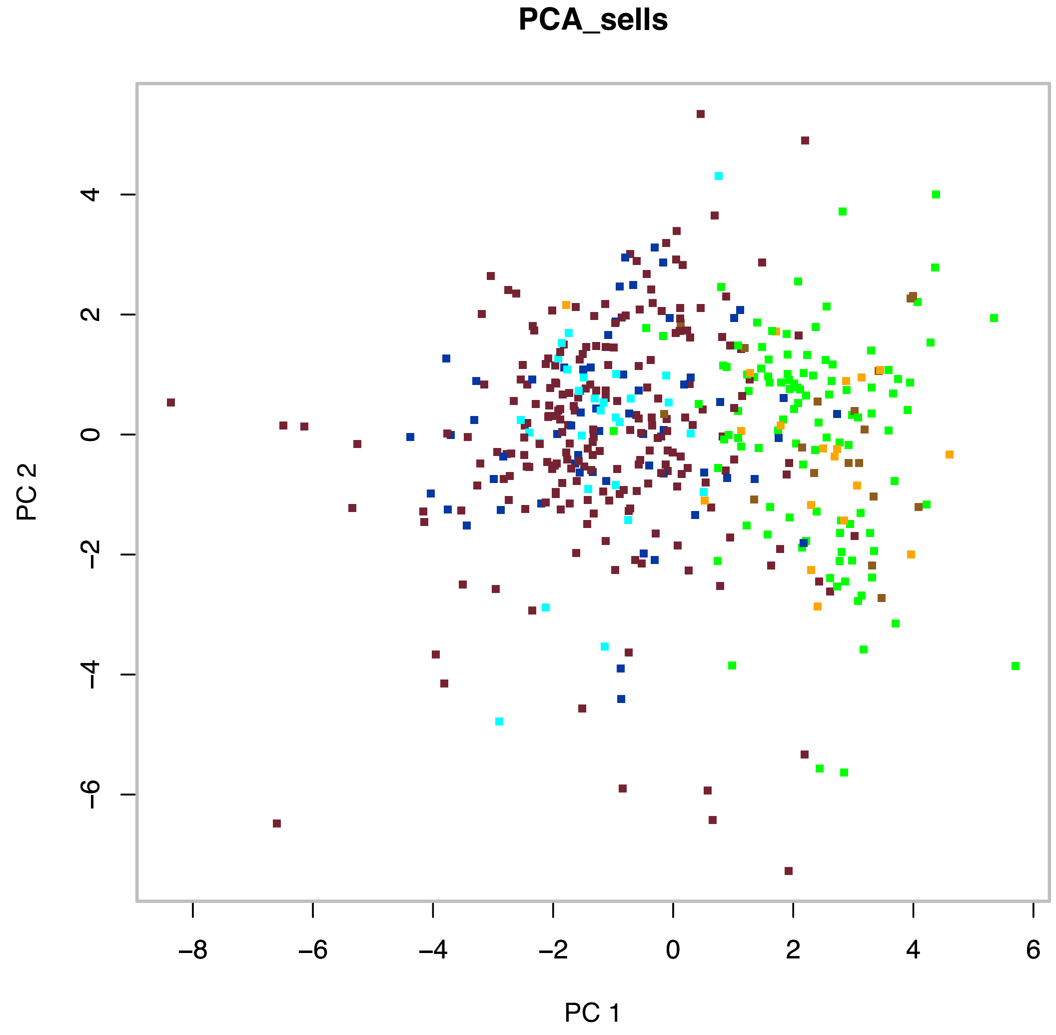


Customer 440

PCA after log transformation

Colors show the **Channel-region labels**

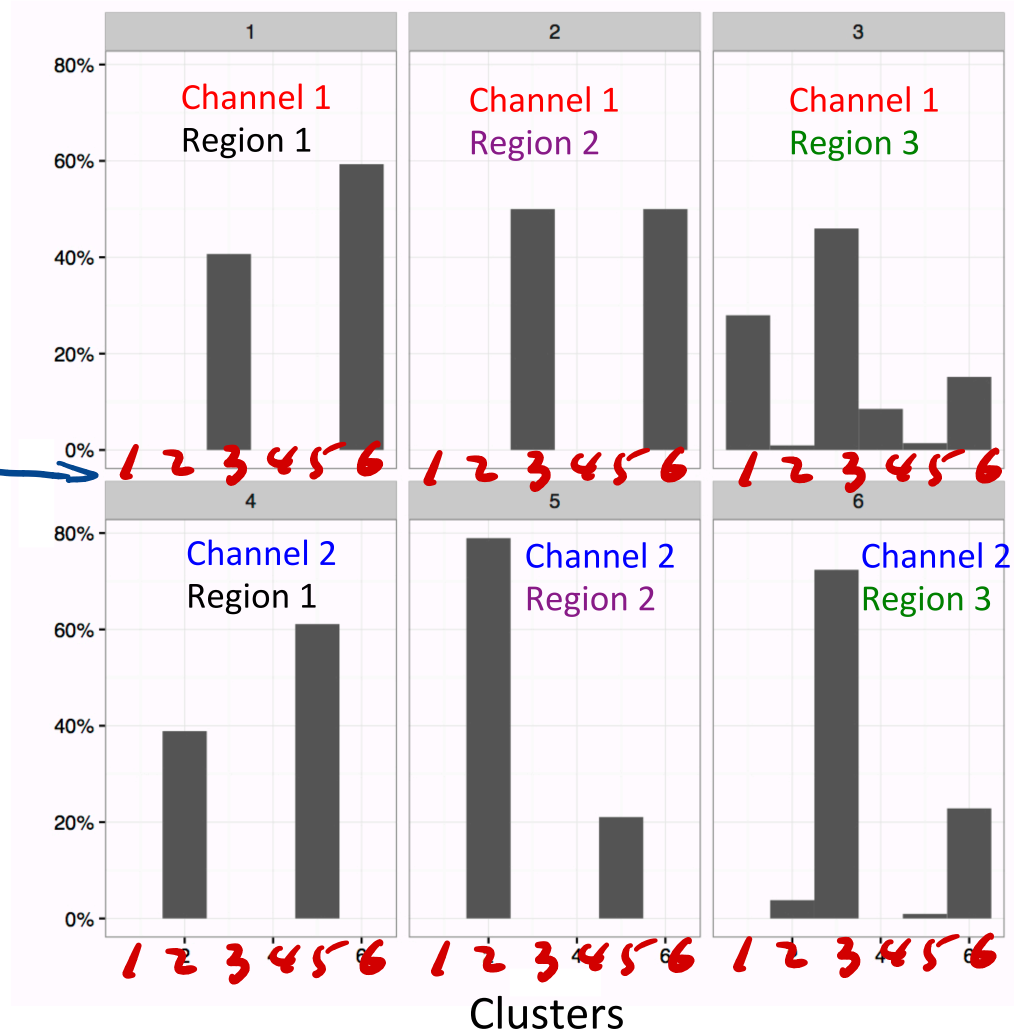
Channels differ a lot



Cluster center histogram of the Portugal grocery spending data

- For each channel/ region, we make a histogram of customers that map to each of the **6 cluster centers**.

- **What do you see?**

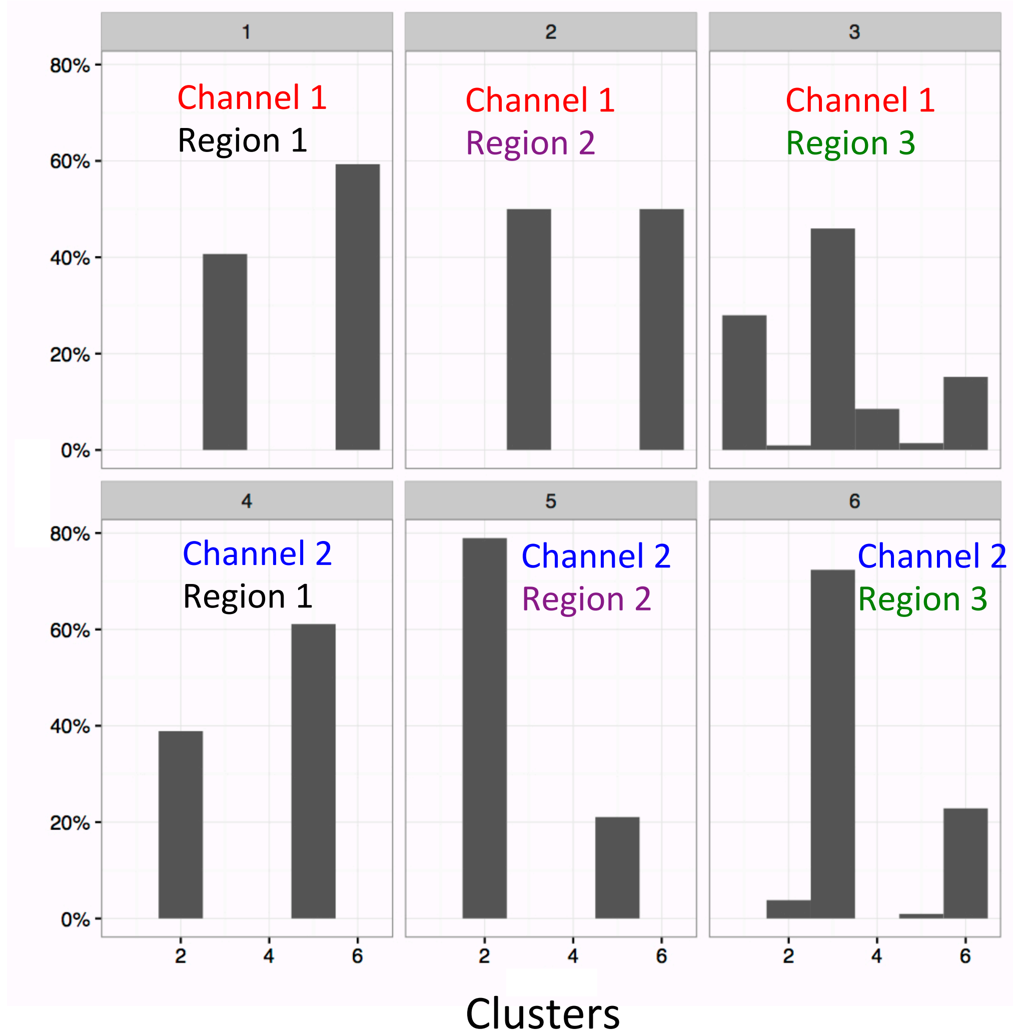


Channel1: Horeca
Channel2: Retail

Region1: Lisbon
Region2: Oporto
Region3: Other

Cluster center histogram of the Portugal grocery spending data

- For each channel/ region, we make a histogram of customers that map to each of the 6 cluster centers.
- **Channels are significantly different!**
- **Region 3 is special**
- **Is it enough to plot the percentage?**

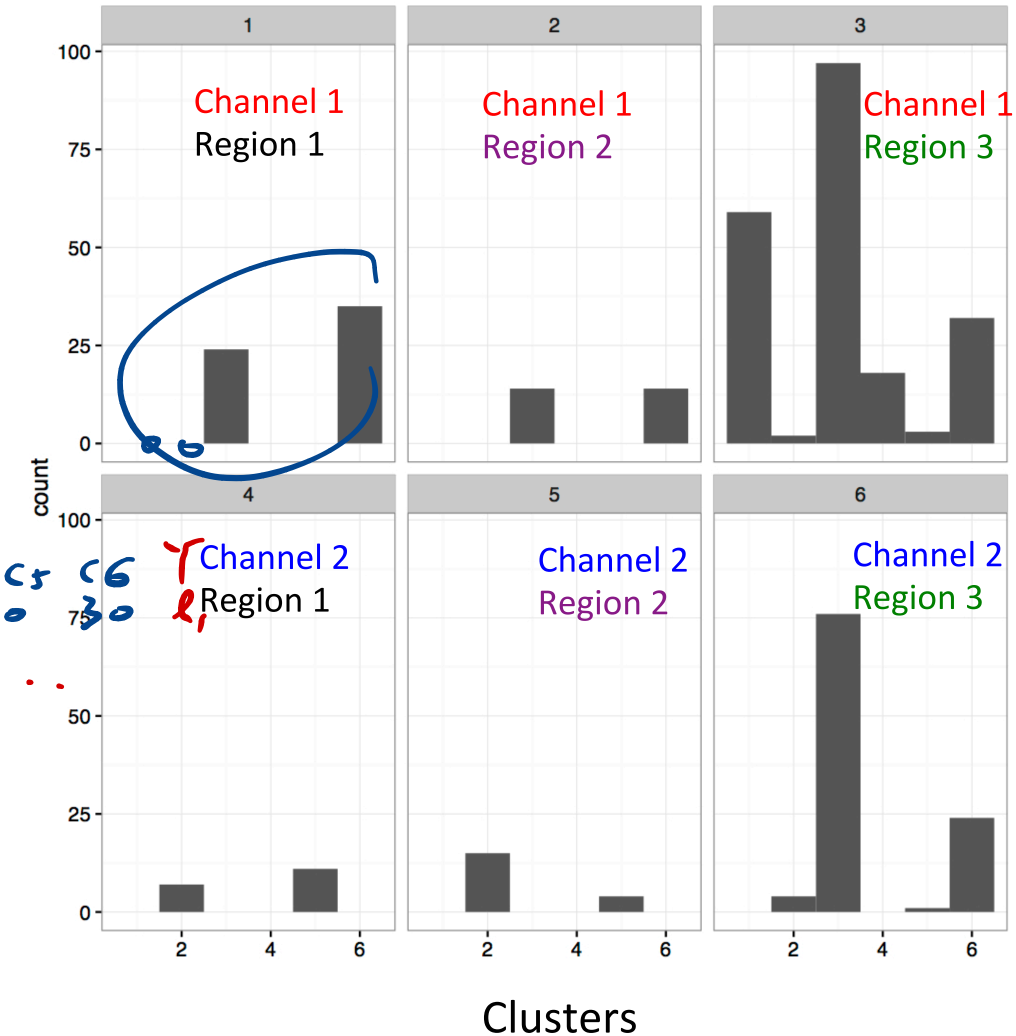


Cluster center histogram of the Portugal grocery spending data

- For each channel/ region, we make a histogram of customers that map to each of the 6 cluster centers.
- **Channels are significantly different!**

c1 c2 c3 c4 c5 c6
0 0 25 0 0 30

- **Region 3 is special**
- **Count matters depending on the purpose**



Q. What can we do with cluster center histograms?

A. investigate the feature patterns of data groups

B. Classify new data with the cluster center histograms.

C. Both A and B.

Markov chain

In a class, students are either up-to-date or behind regarding progress. If a student is up-to-date, the student has 0.8 probability remaining up-to-date, if a student is behind, the student has 0.6 probability becoming up-to-date. Suppose the course is so long that it runs life long, what is the probability any student eventually gets up-to-date?

A 25%

B 50%

$\boxed{C}$ 25% 75%

D 0%

Markov Chain

- Motivation
- Definition of Markov model
- Graph representation – Markov chain
- Transition probability matrix
- The stationary Markov chain
- The pageRank algorithm

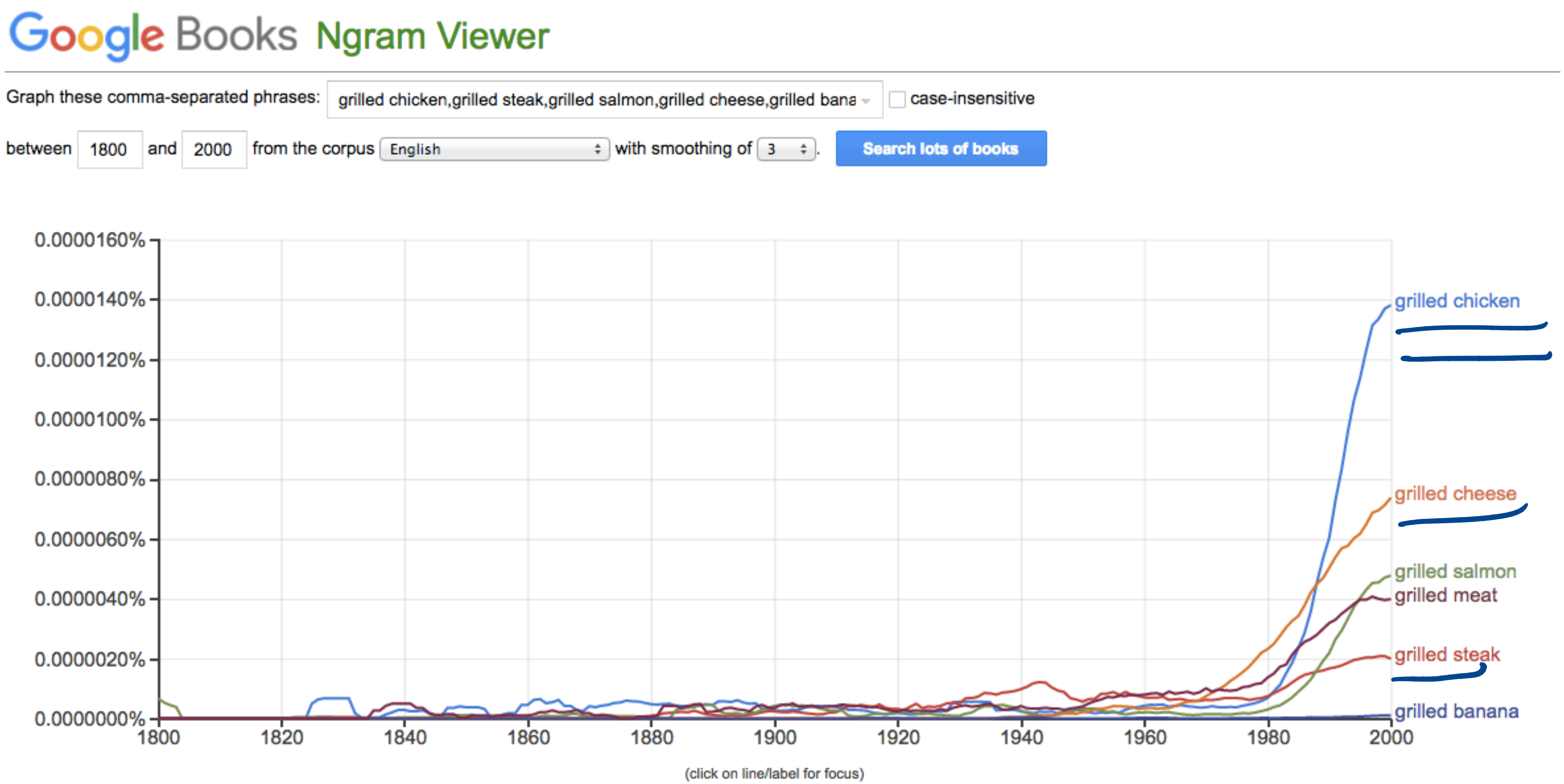
An example of dependent events in a sequence

I had a glass of wine with my grilled ___________

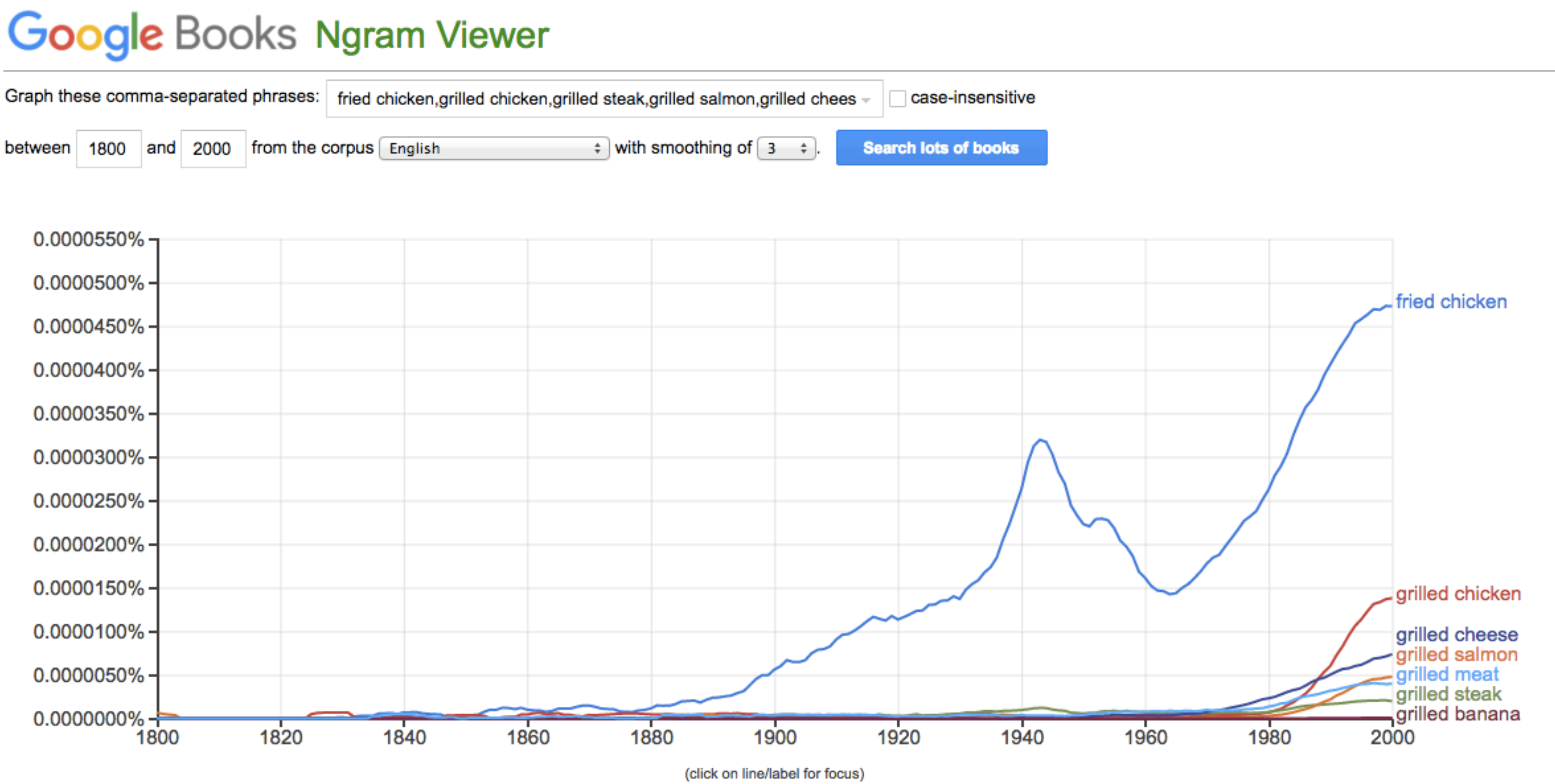
meat
fish
cheese

wood

An example of dependent events in a sequence



An example of dependent events in a sequence



Markov chain

- Markov chain is a process in which outcome of any trial in a sequence is **conditioned by the outcome of the trial immediately preceding, but not by earlier ones.**
- Such dependence is called **chain dependence**

$P(X_n)$ $\downarrow$ $P(X_{n-1})$ X_{n-2} ...

X_t $\downarrow$ X_{t-1} X_{t-2} ...

Andrey Markov (1856-1922)

Markov chain in terms of probability

- Let $X_0, X_1, \ldots$ be a sequence of discrete finite-valued random variables

- The sequence is a Markov chain if the probability distribution X_t only depends on the distribution of the immediately preceding random variable X_{t-1}

$$\boxed{P(X_t|X_0\ldots, X_{t-1}) = P(X_t|X_{t-1})}$$

- If the conditional probabilities (transition probabilities) do **NOT change with time**, it's called **constant Markov chain**.

$$P(X_t|X_{t-1}) = P(X_{t-1}|X_{t-2}) = \ldots = P(X_1|X_0)$$

Coin example

- Toss a fair coin until you see two heads in a row and then stop, what is the probability of stopping after exactly **n** flips?

Random variable

$$\underbrace{* \; * * \; HH}_{n}$$

Geometric

T T T . . . H

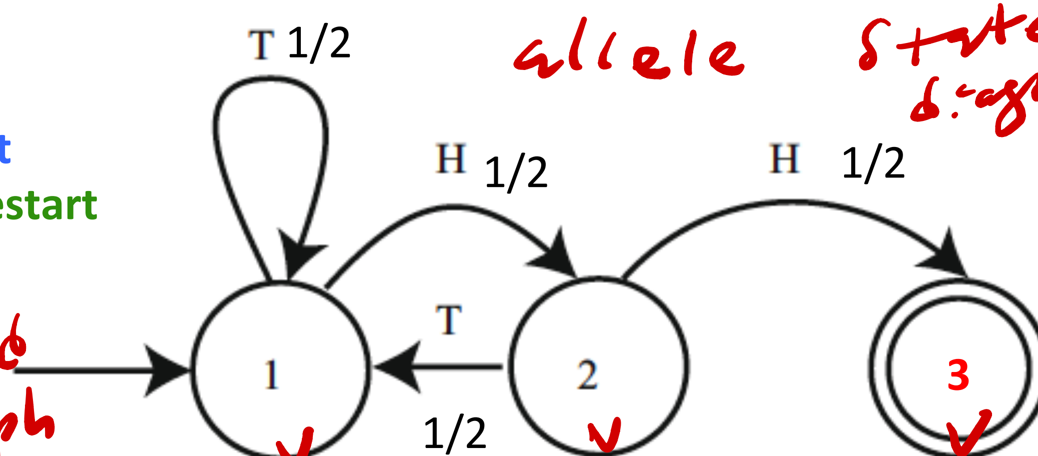
$P(n = n_0) = ?$

allele

state diagram

1 -> Start or just had tail/restart
2 -> had one head after start/restart
3 -> 2heads in a row/Stop

directed graph



N =	1	2	3	4	5	6
Trials	T	T	H	T	H	H
$X_N =$	X_1	X_2	X_3	X_4	X_5	X_6
State	1	1	2	1	2	3

Markov property:

Given the current state
the past doesn't matter

$$p_{ij} = P(X_{n+1}=j \mid X_n = i)$$

$$= P(X_{n+1}=j \mid X_n=i, \boxed{X_{n-1}=? \quad - \cdots \quad X_0=?})$$

This part can be any!!

The model helps form recurrence formula

- Let p_n be the probability of stopping after **n** flips

$$p_1 = 0 \quad p_2 = 1/4 \quad p_3 = 1/8 \quad p_4 = 1/8 \quad \ldots$$

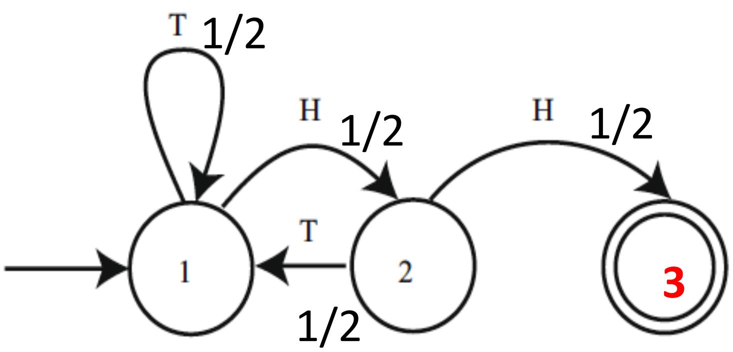
HH

* H H
T H H

* * H H
T T H H
H T H H

$P(n = n_0) =$

$n \uparrow$



The model helps form recurrence formula

- Let p_n be the probability of stopping after **n** flips

$$p_1 = 0 \quad p_2 = 1/4 \quad p_3 = 1/8 \quad p_4 = 1/8 \quad \ldots$$

n

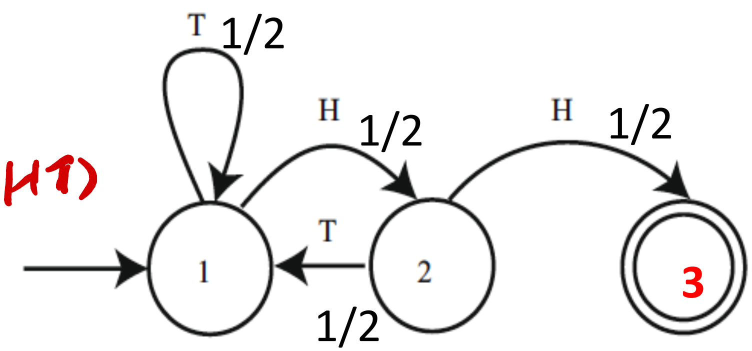
- If $n > 2$, there are two ways the sequence starts
  - Toss T and finish in n-1 tosses
  - Or toss HT and finish in n-2 tosses

T n-1 n

HT n-2

- So we can derive a recurrence relation

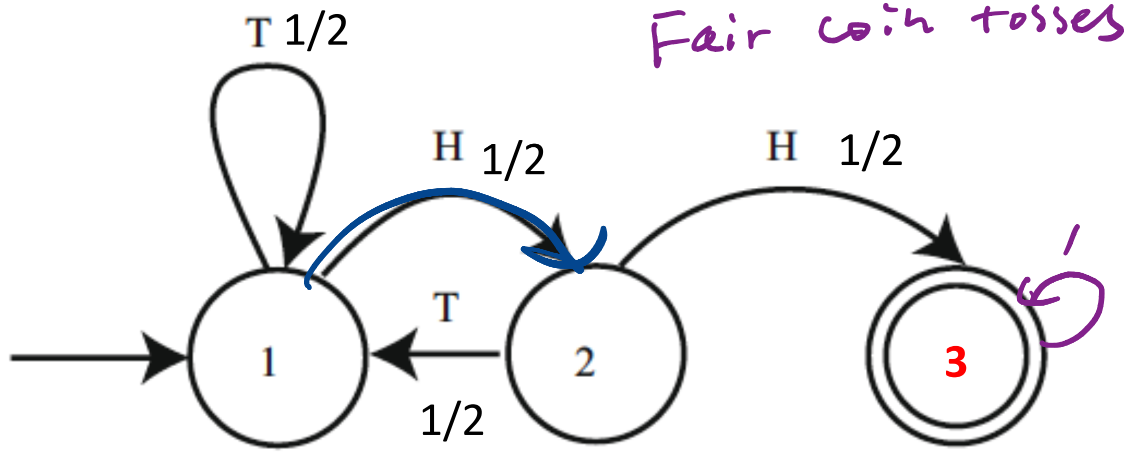
$$p_n = \underbrace{\frac{1}{2}}_{P(T)} \underbrace{p_{n-1}}_{P(n-1|T)} + \underbrace{\frac{1}{4}}_{P(HT)} \underbrace{p_{n-2}}_{P(n-2|HT)}$$



Transition probability btw states

States	1	2	3
1	$\frac{1}{2}$	$\frac{1}{2}$	0
2	$\frac{1}{2}$	0	$\frac{1}{2}$
3	0	0	1

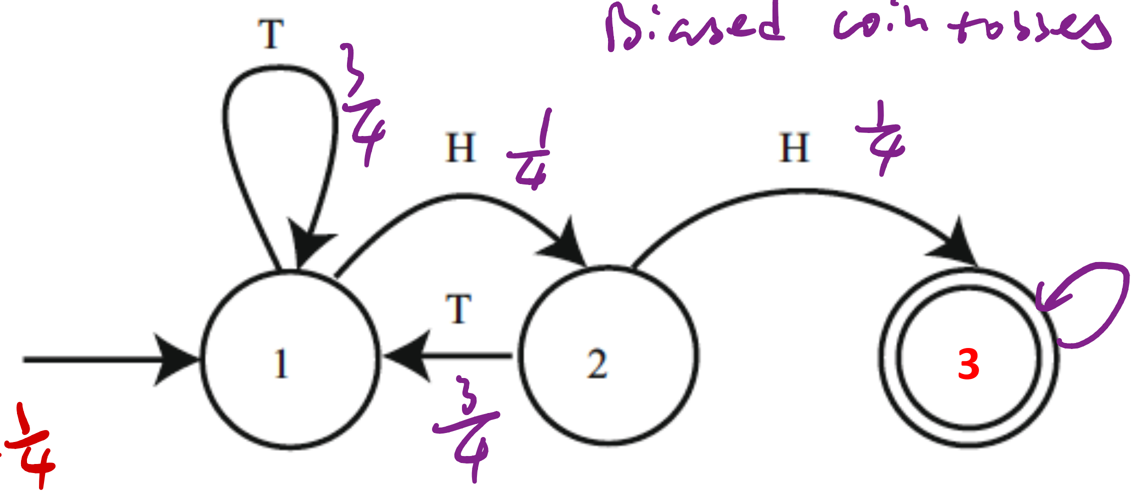
Fair coin tosses



	1	2	3
1	? $\frac{3}{4}$	$\frac{1}{4}$	0
2	$\frac{3}{4}$	0	$\frac{1}{4}$
3	0	0	1

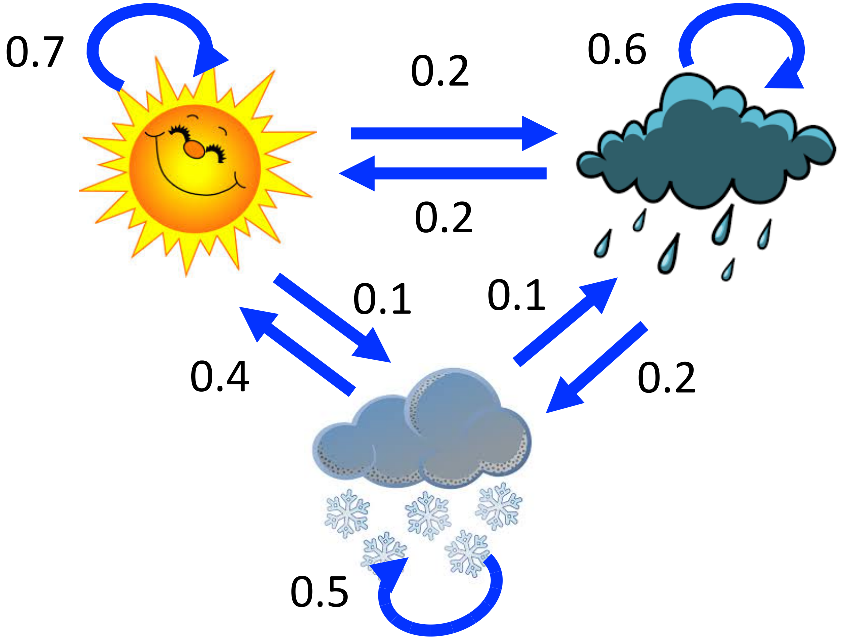
$P(2|1) = \frac{1}{4}$

Biased coin tosses



Transition probability matrix: weather model

- Let's model daily weather as one of the three states (Sunny, Rainy, and Snowy) with Markov chain that has the transition probabilities as shown here.



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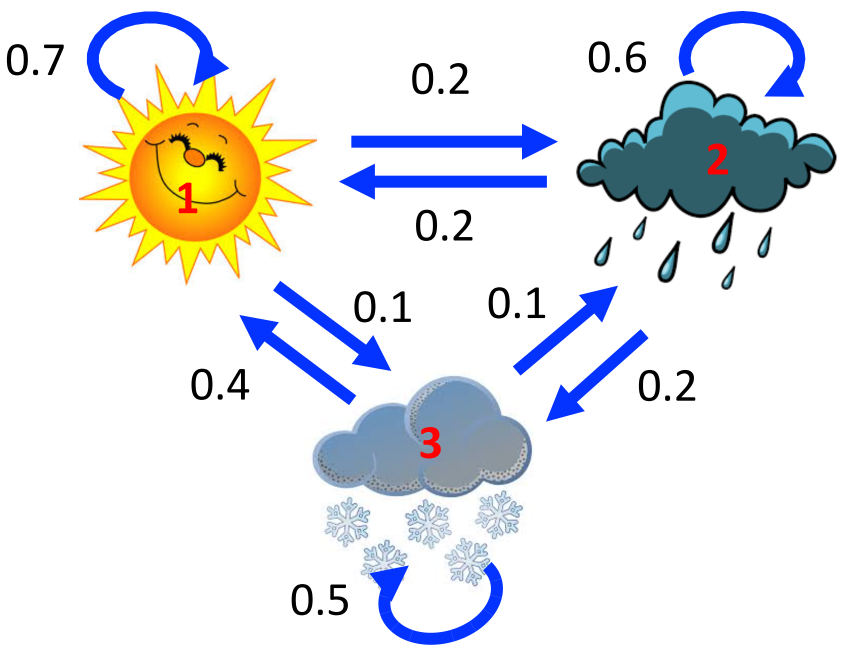

i, the current state at time point t
j, the next state at time point t+1

$$P = \begin{array}{c} \\ Su \\ R \\ Sn \end{array} \begin{array}{c} \begin{array}{ccc} \text{Sunny} & \text{Rainy} & \text{Snowy} \end{array} \\ \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{Sunny} \\ \text{Rainy} \\ \text{Snowy} \end{array}$$

The transition probability matrix

Q: The transition probabilities for a node sum to 1

A. Yes.

B. No.

Only the row sum is 1, that is: the probabilities associated with outgoing arrows sum to 1.

P is a transition prob. matrix

$$\pi_0 = \begin{array}{ccc} \text{Sunny} & \text{Rainy} & \text{Snowy} \\ \left[0 \right. & 1 & \left. 0 \right] \end{array} \qquad t$$

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$$

P(Snowy) for next day? $t+1$

$$\pi_1 = \pi_0 P$$

$$= [0 \quad 1 \quad 0] \begin{bmatrix} \quad \end{bmatrix} = \begin{array}{ccc} \text{Sunny} & \text{Rainy} & \text{Snowy} \\ \left[\quad \right. & & \left. 0.2 \right] \end{array}$$

Additional References

- Robert V. Hogg, Elliot A. Tanis and Dale L. Zimmerman. "Probability and Statistical Inference"
- Kelvin Murphy, "Machine learning, A Probabilistic perspective"

See you next time

See You!