Recap

- (Ch 6) Drawing conclusions from a sample of the population
  - Calculating the standard error of the sample mean
  - Constructing confidence intervals around the sample mean

Today

- (Ch 6) Drawing conclusions from a sample of the population
  - Calculating the standard error for other statistics (e.g. sample median)
  - Constructing confidence intervals for other statistics (e.g. sample median)
- (Ch 7) Assessing the significance of evidence against a hypothesis

Calculating standard error of sample mean

- Calculate the unbiased estimate of the population standard deviation

$$\text{stdunbiased}(\{x\}) = \sqrt{\frac{1}{N-1} \sum_{x_i \epsilon \text{sample}} (x_i - \text{mean}(\{x_i\}))^2}$$

- The standard error is estimated as

$$\text{stderr}(\{x\}) = \frac{\text{stdunbiased}(\{x\})}{\sqrt{N}}$$

Constructing confidence intervals for $N \geq 30$

- 68% confidence interval

$$[\text{mean}(\{x\}) - \text{stderr}(\{x\}), \text{mean}(\{x\}) + \text{stderr}(\{x\})]$$

- 95% confidence interval

$$[\text{mean}(\{x\}) - (2)\text{stderr}(\{x\}), \text{mean}(\{x\}) + (2)\text{stderr}(\{x\})]$$

- 99% confidence interval

$$[\text{mean}(\{x\}) - (3)\text{stderr}(\{x\}), \text{mean}(\{x\}) + (3)\text{stderr}(\{x\})]$$

What does a 99% confidence interval mean?

- For 99% of the samples, if you construct the confidence interval in this way, the population mean will lie within the interval
- It does **not** mean that the population mean lies in the interval with probability 99%

↳ This does not make sense in a frequentist point-of-view

Standard error: midterm grading example

The realized sample of scores is $\{120, 130, 140, 140, 150\}$ with $N = 5$

$$\text{mean}(\{120, 130, 140, 140, 150\}) = 136$$

$$\text{stdunbiased}(\{x\})$$

$$= \sqrt{\frac{(120 - 136)^2 + (130 - 136)^2 + 2(140 - 136)^2 + (150 - 136)^2}{5 - 1}} = 11.4$$

$$\text{stderr}(\{x\}) = \frac{11.4}{\sqrt{5}} = 5.1$$

So we estimate the population mean as 136 with standard error 5.1

Standard error: election polling example

		DATES	POLLSTER	SAMPLE	RESULT				NET RESULT
U.S. House	• IL-12	SEP 26-27	DCCC Targeting Team*	574 LV	Kelly	44%	46%	Bost	Bost +2
	• IL-12	SEP 26-27	DCCC Targeting Team*	574 LV	Kelly	41%	42%	Bost	Bost +1
Governor	• Ill.	SEP 24-29	Southern Illinois University	715 LV	Pritzker	49%	27%	Rauner	Pritzker +22

Source: fivethirtyeight.com

$$\text{stdunbiased}(\{x\}) = \sqrt{\frac{715(0.49)(1-0.49)^2 + 715(0.51)(0-0.49)^2}{715-1}} = 0.50$$

who select Pritzker

who don't

$$\text{stderr}(\{x\}) = \frac{0.5}{\sqrt{715}} = 0.019$$

So we estimate the population mean as 49% with standard error ~~19%~~ 1.9%

Confidence interval: election polling example

		DATES	POLLSTER	SAMPLE	RESULT				NET RESULT
U.S. House	• IL-12	SEP 26-27	DCCC Targeting Team*	574 LV	Kelly	44%	46%	Bost	Bost +2
	• IL-12	SEP 26-27	DCCC Targeting Team*	574 LV	Kelly	41%	42%	Bost	Bost +1
Governor	• Ill.	SEP 24-29	Southern Illinois University	715 LV	Pritzker	49%	27%	Rauner	Pritzker +22

Source: fivethirtyeight.com

We estimated the population mean as 49% with standard error 1.9%

The 99% confidence interval for Pritzker's vote percentage is

$$[49\% - (3)1.9\%, 49\% + (3)1.9\%] = [43.3\%, 54.7\%]$$

Confidence intervals for other statistics

- The **bootstrap** is a method to construct confidence intervals for other statistics (besides the sample mean) for which we cannot derive analytical expressions

- Bootstrapping a confidence interval for the sample median given a sample of N items
  - Create a bootstrap replicate by sampling N items from the original sample uniformly and **with replacement**
  - Calculate the sample median of the bootstrap replicate
  - Repeat the two steps above a large number of times
  - Plot the histogram of the sample medians and construct a confidence interval

Bootstrap median: midterm grading example

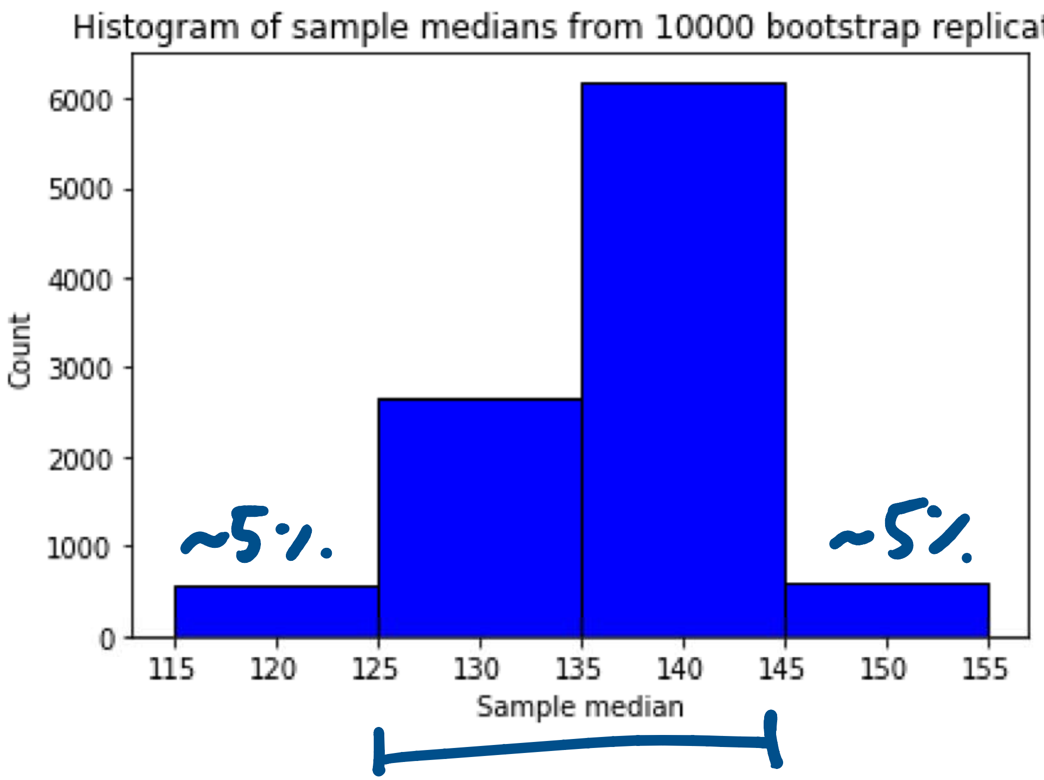
The realized sample of scores is $\{120, 130, 140, 140, 150\}$ with $N = 5$

$$\text{median}(\{120, 130, 140, 140, 150\}) = 140$$

Bootstrap replicate	Sample median
$\{150, 150, 130, 130, 140\}$	140
$\{120, 120, 140, 140, 120\}$	120
$\{130, 140, 140, 150, 140\}$	140
$\{150, 120, 130, 130, 140\}$	130
⋮	⋮

Bootstrap median: midterm grading example

Repeat a total of 10000 times and plot a histogram of sample medians



90% confidence interval for sample median

The scientific method

- Form a **hypothesis** about some phenomenon

- Design an experiment and collect data

- Reject the hypothesis if it is contradicted by the data

Hypothesis testing: election polling example

Hypothesis: Pritzker’s vote percentage is 53%

Experiment

		DATES	POLLSTER	SAMPLE	RESULT				NET RESULT
U.S. House	• IL-12	SEP 26-27	DCCC Targeting Team*	574 LV	Kelly	44%	46%	Bost	Bost +2
	• IL-12	SEP 26-27	DCCC Targeting Team*	574 LV	Kelly	41%	42%	Bost	Bost +1
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Should we reject the hypothesis based on this data?

Fraction of "less extreme" samples

- Assuming that the hypothesis is true, what fraction of samples would have had sample means less extreme than what we observed?

- Define a test statistic $g = \frac{(\text{sample mean}) - (\text{hypothesised value})}{\text{standard error}}$

  0.49 0.53

  0.019

- If $N \geq 30$, we can say g comes from a standard normal distribution

- So, the fraction of "less extreme" samples $f = \frac{1}{\sqrt{2\pi}} \int_{-|g|}^{|g|} \exp\left(-\frac{x^2}{2}\right) dx$

P-value: fraction of “more extreme” samples

- It is conventional in science to report the p-value of an experiment

$$p = 1 - f = 1 - \frac{1}{\sqrt{2\pi}} \int_{-|g|}^{|g|} \exp\left(-\frac{x^2}{2}\right) dx$$

- So, a p-value is the fraction of samples that would have had sample means **more** extreme than what we observed, assuming that the hypothesis is true

- By convention, if the p-value < 0.05, we should reject the hypothesis

P-value: election polling example

- Hypothesis: Pritzker's vote percentage is 53%
- Recall that we calculated sample mean 49% and standard error 1.9%
- So the test statistic $g = \frac{49-53}{1.9} = -2.11$
- And the p-value tells us to reject the hypothesis

$$p = 1 - \frac{1}{\sqrt{2\pi}} \int_{-2.11}^{2.11} \exp\left(-\frac{x^2}{2}\right) dx = 0.035 < 0.05$$

The use and misuse of p-values

- P-values in scientific practice
  - Scientists are usually trying to reject the null hypothesis, the hypothesis that there is no special phenomenon and the data are just random noise
  - So $p < 0.05$ means there may be an interesting phenomenon and it has been the standard for publication in many fields

- What's wrong with using p-values in this way?
  - Rejecting the null hypothesis doesn't mean that the proposed alternative hypothesis is true
  - $p < 0.05$ is arbitrary and gives a 1-in-20 chance of false positives
  - It encourages p-value hacking and has contributed to the replication crisis