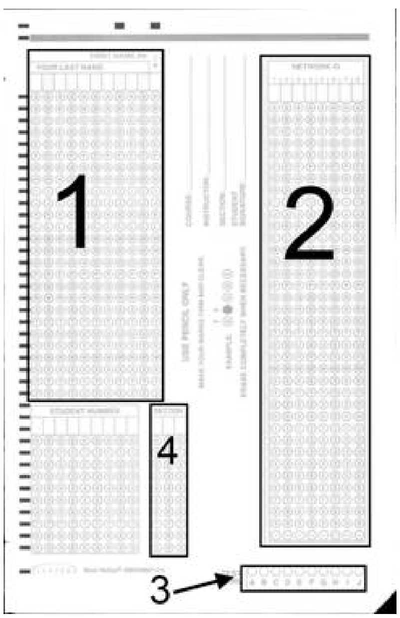
CS357 **Spring 2012**

Exam #2 A Thursday, April 12th

READ and complete the following:

- Bubble your Scantron only with a No. 2 pencil.
- On your Scantron (shown in the figure below), bubble :

  1. Your Name
  2. Your NetID
  3. Form letter "**A**"



- No electronic devices or books are allowed while taking this exam. However, you may use a "cheat sheet" - a single sheet of size 8.5" x 11" or smaller.
- Please fill in the most correct answer on the provided Scantron sheet.
- We will not answer any questions during the exam.
- Each question has only ONE correct answer.
- You must stop writing when time is called by the proctors.
- Hand in both these exam pages and the Scantron.
- DO NOT turn this page UNTIL the proctor instructs you to.

For the next 2 questions, use the following matrix:

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & 0 & 4 \\ 6 & 5 & 0 \end{bmatrix}.$$

1. Consider CSR (Compressed Sparse Row), what is AA?

(a) $[3 \quad 3 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6]$

(b) $[3 \quad 3 \quad 2 \quad 1 \quad 3 \quad 4 \quad 6 \quad 5]$

(c) $[0 \quad 2 \quad 1 \quad 3 \quad 0 \quad 4 \quad 6 \quad 5 \quad 0]$

(d) $[2 \quad 1 \quad 3 \quad 4 \quad 6 \quad 5]$

2. Consider CSR (Compressed Sparse Row), what is IA?

(a) $[2 \quad 3 \quad 1 \quad 3 \quad 1 \quad 2]$

(b) $[1 \quad 1 \quad 2 \quad 2 \quad 3 \quad 3]$

(c) $[2 \quad 3 \quad 6 \quad 7]$

(d) $[1 \quad 3 \quad 5 \quad 7]$

3. Which of the following statements is **True** concerning the condition number (κ) of a matrix ?

(a) The condition number of a matrix, for some norms, can be negative.

(b) The condition number of the permutation matrix P, for the 1-norm, is NOT equal to 1.

(c) $\kappa(nA) = n * \kappa(A)$, for any norm, where n is any integer

(d) $\kappa(I) = 1$, for the 1-norm, where I is the identity matrix.

4. Which matrix has the following MSR (Modified CSR) representation?

$$AA = [3 \quad 5 \quad 6 \quad * \quad 2 \quad 1 \quad 4 \quad 7]$$

$$JA = [5 \quad 7 \quad 8 \quad 9 \quad 2 \quad 3 \quad 1 \quad 1]$$

(a) $\begin{bmatrix} 3 & 4 & 7 \\ 2 & 5 & 0 \\ 1 & 0 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 0 \\ 7 & 0 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 2 & 1 \\ 4 & 5 & 0 \\ 7 & 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 6 & 4 & 7 \\ 2 & 5 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

5. Consider

$$A = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}.$$

Using Jacobi's iterative method to approximate the solution $Ax = b$ with a starting guess of $x_0 = [1, 0, 1]^T$, what is x_1?

(a) $[1, 4, 1]^T$

(b) $[0, 1, 1]^T$

(c) $[0, 2, 1]^T$

(d) $[1, 1, 0]^T$

6. The lower triangular matrix named L formed by performing the Cholesky factorization of matrix $A = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$ looks like $L = \begin{bmatrix} p & 0 \\ q & r \end{bmatrix}$.

What is the value of $p + q + r$?

(a) 13

(b) 5

(c) 7

(d) 6

7. What is cost of using Gaussian Elimination in solving the linear system $Ax = b$ where A is an m-band, $n \times n$ matrix?

(a) $O(n^2)$

(b) $O(m^2)$

(c) $O(m^2 n)$

(d) $O(m^2 n^2)$

8. Which of the following statement is **NOT** always true about norms $\|x\|$ for any given vector x ?

(a) $\|x\| > 0$

(b) $\|ax\| = |a|.\|x\|$

(c) $\|x + y\| \leq \|x\| + \|y\|$

9. What is the condition number of a 3 by 3 diagonal matrix with diagonal elements -1 , 4 and -7 ?. Hint: try using the 1-norm.

(a) 0

(b) -4/7

(c) 7/4

(d) 7

10. What is the 2-norm of the following matrix?

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ -\sqrt{2} & 1 \end{bmatrix}$$

(a) 0

(b) 1

(c) $\sqrt{2}$

(d) 2

11. What is the form of $A^T A$ in the normal equation?

(a) diagonal

(b) symmetric

(c) orthonormal

(d) orthogonal

12. What are the singular values of the following matrix?

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$$

(a) $\sqrt{10}$ and 0

(b) 10 and 0

(c) 3 and 0

(d) $\sqrt{3}$ and 0

13. Which of the following is **True** concerning the solution to the normal equations $A^T Ax = A^T b$?

(a) The normal equations always has a unique solution.

(b) Normal equations has a unique solution when the matrix A has full rank.

(c) Solving the normal equations is always the best way to solve a linear least squares problem.

(d) The condition number of the matrix $A^T A$ in the normal equations is always equal to one.

14. Consider $y = 2{*}x_1{*}t{+}3{*}x_2{*}t^2$, and $(t_1, y_1) = (1, 2), (t_2, y_2) = (4, 7), (t_3, y_3) = (6, 12)$ which of the Matrix is Vandermonde matrix? That is, which is the matrix A where $Ax \approx b$, and $x = [x_1 \;\; x_2]^T$?

(a) $\begin{bmatrix} 2 & 3 \\ 8 & 12 \\ 12 & 18 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 8 & 48 \\ 12 & 108 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 4 & 16 \\ 6 & 36 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 \\ 8 & 12 \\ 36 & 108 \end{bmatrix}$

15. Which of the following matrices is **NOT** orthogonal?

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

16. Let Q be the matrix as given below.

$$Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & q_1 \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & q_2 \\ \frac{1}{\sqrt{3}} & 0 & q_3 \end{bmatrix}$$

Find the values of (q_1, q_2, q_3) for which Q is an orthogonal matrix.

(a) $(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}})$

(b) $(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}})$

(c) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}})$

(d) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$

17. Matrix A is defined below. Eigenvalues of AA^T were found to be λ_1=200 and λ_2=50. Find the left singular vectors of A. Hint: the left singular vectors are columns of the matrix U where $A = USV^T$.

$$\text{A} = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix}$$

(a) $u_1 = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}, u_2 = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

(b) $u_1 = \begin{bmatrix} \frac{-1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{bmatrix}, u_2 = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{bmatrix}$

(c) $u_1 = \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \end{bmatrix}, u_2 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \end{bmatrix}$

(d) $u_1 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, u_2 = \begin{bmatrix} \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$

18. If the square matrix Q is orthogonal, then what is the value of $\|Q\|_2$?

(a) 0

(b) 1

(c) 2

(d) $+\infty$

19. What is the 1-norm of the matrix A?

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

(a) 1

(b) 2

(c) 3

(d) 6

20. Consider the matrix $A = \begin{bmatrix} 1 & \rho \\ -\rho & 1 \end{bmatrix}$, under which conditions is the Gauss-Seidel method guaranteed to converge with this matrix?

(a) $|\rho| < 1$

(b) $|\rho| > 1$

21. One limitation of using the normal equations for solving a least squares problem is the fact that $\kappa(A^T A) = (\kappa(\text{A}))^2$ where $\kappa()$ denotes the condition number.

(a) True

(b) False

22. Suppose that the non-singular square matrix A can be factored as $A = QR$, where Q is orthogonal and R is upper triangular. Then to solve $Ax = b$ we would

(a) Compute $c = Q^T b$ and then use back substitution to solve $Rx = c$.

(b) Use back substitution to solve $Rx = b$ and then compute Qb.

(c) Compute $c = Q^T b$ and then use forward substitution to solve $Rx = c$.

(d) Use forward substitution to solve $Rx = b$ and then compute $Q^T b$.

23. Let the $n \times n$ matrix named V have columns denoted by v_i for $i = 1, \ldots, n$ and an $n \times 1$ vector named a have values a_i for $i = 1, \ldots, n$ then the sum $\sum_{i=1}^{n} a_i * v_i$ can be written in which of the following equivalent way?

(a) $V^T a^T$

(b) aV

(c) $a^T V^T$

(d) Va

24. Given the matrix A shown below, what is the L in the Cholesky factorization of A?

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 10 \end{bmatrix}$$

(a) $L = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$

(c) $L = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

(d) $L = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

25. Let A be an $n \times n$ non-singular matrix. If $A = QR$ where Q is an $n \times n$ orthogonal matrix and R is an $n \times n$ upper triangular matrix then which of the equalities below are equivalent to $A^T Ax = A^T b$?

(a) $Rx = Qb^T$

(b) $Rx = Q^T b$

(c) $Qx = R^T b$

(d) $Q^T x = Rb^T$