CS357 **Spring 2011**
Exam #1 B **Thursday, February 24th**

No computers, calculators, or books allowed.
You may take with you a "cheat sheet" - a single sheet of size 8.5" x 11" or smaller.
You may also use the back pages of this exam for scratch paper.

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1. True or False (*) All machine numbers in IEEE-754 are equally spaced.

2. What is $[1, 2] \times [3, 4]$, where $\times$ is interval multiply?

(a) $[2, 4]$
(b) $[1, 8]$
(c) $[3, 8]$(*)
(d) $[6, 8]$

3. The first three terms of the Taylor expansion of e^x are

$$e^x \approx 1 + x + \frac{x^2}{2}.$$

What is the relative error in approximating e^1 with the above series?

(a) $\frac{\left|\frac{5}{2}-e\right|}{2}$
(b) $\frac{\left|\frac{5}{2}-e\right|}{e}$(*)
(c) $\left|\frac{5}{2} - e\right|$
(d) $\frac{\left|\frac{5}{2}-e\right|}{\frac{5}{2}}$

4. Consider the following MATLAB code:

```
x = 2^(-100);
while 2 + x == 2
     x = 2*x;
end
```

What will happen when this code is run?

(a) The loop will run forever.
(b) The loop will terminate. The final value of `x` is 2^{-100}.
(c) The loop will terminate. The final value of `x` is the IEEE-754 ϵ_m, i.e. machine precision.
(d) The loop will terminate. The final value of `x` is twice the IEEE-754 ϵ_m, i.e. twice machine precision.(*)

5. Let $f(h) = h + h^2 + h^3 + h^4$. Which of the following describes the asymptotic behavior of $f(h)$ as $h \to 0$?

(a) $\mathcal{O}(h)$(*)
(b) $\mathcal{O}(h^2)$
(c) $\mathcal{O}(h^3)$
(d) $\mathcal{O}(h^4)$

6. Let $f(n) = n + n^2 + n^3 + n^4$. Which of the following describes the asymptotic growth of $f(n)$ as $n \to \infty$?

(a) $\mathcal{O}(n)$
(b) $\mathcal{O}(n^2)$
(c) $\mathcal{O}(n^3)$
(d) $\mathcal{O}(n^4)$ (*)

7. Using a Taylor series approximation to $f(x) = (\cos(x))^2$ with the first two non-zero terms, about the point $x = 0$, what is the approximation to $f(0.1)$?

(a) 1.0
(b) 0.99 (*)
(c) 0.98
(d) 1.01

8. For an $n \times n$ system of linear equations, which of the following statement can **never** be **true**?

(a) There is no solution
(b) There is exactly one solution
(c) There are exactly two solutions(*)
(d) There are infinitely many solutions

9. Which one of the following statements regarding the following system is **false**?

$$Ax = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = b$$

(a) With $b = [6 \ \ 18]^T$, there are infinitely solutions
(b) With $b = [6 \ \ 19]^T$, there are no solutions
(c) There is exactly one solution regardless of value of b(*)

10. Let $A = \begin{bmatrix} \delta & 1 \\ 1 & -\delta \end{bmatrix}$ where δ is a real number. Which one of the following is **false**?

(a) The inverse of A is $\begin{bmatrix} \dfrac{\delta}{1+\delta^2} & \dfrac{1}{1+\delta^2} \\ \dfrac{1}{1+\delta^2} & \dfrac{-\delta}{1+\delta^2} \end{bmatrix}$

(b) The matrix A is nonsingular for all real δ

(c) The LU factorization of A without pivoting exists for $\delta = 0$ (*)

(d) The LU factorization of A without pivoting does not exist for $\delta = 0$

11. Having obtained LU factorization of A, $Ax = b$ becomes $LUx = b$, it can be solved by

(a) forward-substitution of an upper triangular system followed by backward-substitution of a lower triangular system

(b) backward-substitution of a lower triangular system followed by forward-substitution of an upper triangular system

(c) forward-substitution of a lower triangular system followed by backward-substitution of an upper triangular system (*)

(d) backward-substitution of an upper triangular system followed by forward-substitution of a lower triangular system

12. Let

$$A = \begin{bmatrix} 1 & 1+\alpha \\ 1-\alpha & 1 \end{bmatrix}$$

Which one of the following statements is **false**?

(a) Using exact arithmetic for computing the determinant, the determinant of A is α^2

(b) In floating-point arithmetic, if $|\alpha| < \sqrt{\epsilon_m}$, then the computed value of the determinant will be zero. (where ϵ_m denotes the machine epsilon)(*)

(c) When we compute the LU factorization of A, $L = \begin{bmatrix} 1 & 0 \\ 1-\alpha & 1 \end{bmatrix}$

(d) When we compute the LU factorization of A, $L = \begin{bmatrix} 1 & 1+\alpha \\ 0 & \alpha \end{bmatrix}$

13. Compute the LU factorization of A. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

Which one of the following is **false**?

(a) The elementary elimination matrices $M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, $M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}$

(b) $U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $L_1 = M_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, $L_2 = M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix}$ (*)

(d) $L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 2/3 & 1 \end{bmatrix}$

14. Given the LU factorization of the A as shown below compute the determinant of A.

$$A = \begin{bmatrix} 5 & -5 & 10 \\ 2 & 0 & 8 \\ 1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.2 & 1 & 1 \end{bmatrix} * \begin{bmatrix} 5 & -5 & 10 \\ 0 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix}$$

(a) det(A) = 0

(b) det(A) = 1

(c) det(A) = 10

(d) det(A)= -10 (*)

15. What is the ∞-norm of the vector

$$\begin{bmatrix} -2 \\ 0 \\ -5 \end{bmatrix}$$

(a) 0

(b) 2

(c) 5 (*)

(d) 7

16. Using Gaussian Elimination with partial pivoting to solve the following matrix problem, what is the first pivot position?

$$Ax = \begin{bmatrix} 1 & 1 & 2 & -1 \\ 1 & 0 & -4 & 1 \\ 1 & 2 & 8 & 1 \\ -2 & 1 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = b$$

- **(a)** Row 1, Column 1 (*)
- **(b)** Row 2, Column 1
- **(c)** Row 2, Column 2
- **(d)** Row 3, Column 3

17. True or False (*) If condition number for a function $G : \mathbb{R} \to \mathbb{R}$ is large then $G(x)$ is insensitive to changes in x.

18. A is an nxn matrix. If A is singular, which of the following statement is **NOT** true?

- **(a)** The inverse of A does not exist.
- **(b)** $Rank(A)$ is less than n.
- **(c)** A solution to $Ax = b$ does not exist.(*)
- **(d)** $Det(A) = 0$.

19. Assume that the bisection method converges to find a root r for a function $f(x)$ in the interval $[0,\ 1]$. How many iterations are required to obtain $|x_n - r| < 0.01$? You are given that $sign(f(0)) \neq sign(f(1))$ and $x_0 = 0.5$.

- **(a)** 4
- **(b)** 6 (*)
- **(c)** 8
- **(d)** 10

20. Applying Newton's method to $f(x) = (x-1)^2$ with initial guess $x_0 = 0$, what is the value of x after **two** iteration?

- **(a)** 0
- **(b)** 1/3
- **(c)** 3/4 (*)
- **(d)** 1

21. Applying Secant's method to $f(x) = (x-1)^2$ with initial guesses $x_0 = 0$ and $x_1 = \frac{1}{2}$, what is the value of x after **one** iteration?

- **(a)** 0
- **(b)** 2/3 (*)
- **(c)** 3/4
- **(d)** 1

22. Assume that we use a new root finding method. For this method, at the nth step, the error is $e_n = 1/n^2$. What is the convergence rate?

(a) Cubic

(b) Super linear

(c) quadratic

(d) Sublinear (*)

23. If $f(100) = 5$ and $f(101) = 5.1$, which of the following is a good estimate for the absolute condition number of $f(x)$ at $x = 100$?

(a) 0.1 (*)

(b) 2

(c) 10

(d) 0.5

24. Consider solving system of equations $x_1 x_2 + 10 = 0$ and $x_2 - 1 = 0$ using Newton's method. For which of the following starting values will this method fail?

(a) $(x_1, x_2) = (1, 2)$

(b) $(x_1, x_2) = (1, 0)$ (*)

(c) $(x_1, x_2) = (1, 1)$

(d) $(x_1, x_2) = (2, 1)$

25. Given the code below, compute the best asymptotic bound for the number of floating point operations based on the value n.

```
for k = 1,...,n-1
  for i = k+1,...,n
    xmult = A(i,k)/A(k,k)
    A(i,k) = xmult
    for j = k+1,...,n
      A(i,j) = A(i,j) - xmult*A(k,j)
    end
    b(i) = b(i) - xmult*b(k)
  end
end
```

(a) $O(n)$

(b) $O(n^2)$

(c) $O(n^3)$(*)

(d) $O(n^4)$